ANALYTICAL HAND FORMULA FOR MEMBER STIFFNESS UNDER CLAMPED ZONE IN BOLTED JOINTS

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ABSTRACT
The quantification of contact area and pressure distribution in a bolted joint is essential information, as it determines the integrity of the joint. Most of Current bolted joint designs are based on approximate solutions of the pressure distribution in complicated hand formulas. The purpose of the present paper is to find a simplified analytic expression for the stiffness of the clamped member. The derivation is based on the principle of superposition. The stiffness of the conical solid member is estimated first and from which the stiffness of the hole material is subtracted. The results obtained were accurate and proved to agree with formulas that are widely used.

KEYWORDS: Bolt, Member, Clamped Zone, Stiffness, Pressure Distribution, Joint, Contact Load.

اشتقاق صيغة تحليلية يدوية لصلابة الأجزاء للمنطقة المضغوطة في وصلات البراغي

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الموجز
أن حساب مساحة التلامس وتوزيع الضغط في وصلات البراغي من الامور الاساسية لضمان وصلات محكمة. أن معظم تصاميم وصلات البراغي تعتمد على حل تقريبي لتوزيع الضغط وبصيغ رياضية مطولة. يعنى هذا البحث باشتقاق صيغة يدوية جديدة ومختصرة لايجاد صلابة الاجزاء المضغوطة بالبراغي . يعتمد الاشتقاق على مبدأ التراكب . حيث يتم حساب صلابة المنطقة المخروطية المضغوطة بالبرغي ثم تطرح منها صلابة المنطقة المثقوبة. ان النتائج المستحصلة كانت دقيقة ومتوافقة مع الصيغ الرياضية المعتمدة حاليا.

NOMENCLATURE

a	hole radius
d	bolt diameter
D	washer diameter
E	elasticity modulus
Kb	bolt stiffness
K_m, K_{member}	member stiffness
K_{mt}	total member stiffness
K_t, K_1,K_2	spring constants
L	Grip length
p	pressure
t	plate thickness

α half apex angle

INTRODUCTION

Bolted joints including bolts and members act like elastic springs under assembly and operating conditions **(Guoqing et al, 2011)**. The bearing and transferring of the initial preload and the operating force are mainly dependent on the stiffness of both the bolts and their corresponding members. Accurate calculation of bolt force in a bolted joint under external loads and temperature changes is a fundamental requirement in many industries. Approximate hand formulas, based on idealized mechanical models, are widely used **(Grant et al, 2010)** .Some researchers use the FEM to determine these stiffnesses **(Grant et al, 2010), (Matthew, 2001), (Joseph, 2009), (Shigley, 2006)** and (Niels**, 2007).** Others combine FEM with fuzzy set theory to solve the uncertainties of bolted joints **(Ibrahim et al, 2005)** The stiffness of the bolt can be estimated rather easily, in contrast to the member stiffness, but with finite element FE and contact analysis, it is possible to find the stiffness of the member **(Niels, 2007).** In the case of many connections and for practical applications, it is not suitable to make a full FE analysis, but a quick hand formula is needed **(Niels, 2007)**. The bolt stiffness can be easily determined by assuming axial loading acting on shaft of regular area along the bolt length. Sometimes the threaded and unthreaded parts of the bolt are treated separately and the total bolt stiffness is determined by combining the two results. The stiffness of the clamped members is rather difficult to solve because the cross section area is varied when moving away from bolt head. The current research focuses on the derivation of a new exact and compact formula for determining member stiffnesses.

THEORY

Ultrasonic techniques have been used to determine the pressure distribution at the member interface **(Ito, 1977)** and **(Marshall, 2006)**. The results show that the pressure stays high out to about 1.5 bolt radii . The pressure however falls off farther away from the bolt. This will lead to the conical loaded member shown in **Figure 1 (Jose, 2000).** Approximate and exact pressure distributions are shown in **Figure 2 (Marshall, 2006). (Osgood, 1979)** reports a range of [$25^0 \le \alpha \le 33^0$] for most combinations; where α is the half apex angle in general cone geometry. In most cases $\alpha = 30^0$.
The stiffness of any two springs acting together is the sum of their corresponding stiffnesses. That mean for the two springs shown in **Figure 3** the total stiffness is $K_t = K_1 + K_2$.
In the case of member stiffness it is possible to get the member stiffness by subtracting the stiffness of the drilled area from the stiffness of the conical solid member as shown in **Figure 4.**

$$K_m = K_{cone} - K_{drilled}$$

where:
K_m: member stiffness
K_{cone}: solid cone stiffness
$K_{drilled}$: stiffness of drilled area.

A- Stiffness of the drilled area:

$$K_{drilled} = \frac{EA}{L} = \frac{E\frac{\pi}{4}d^2}{t} = \frac{E\pi d^2}{4t} \tag{1}$$

B- Stiffness Of The Solid Cone:

For the shaded strip shown,

$$\varepsilon = \frac{dL}{dy} = \frac{\sigma}{E} \tag{2}$$

$$dL = \frac{\sigma}{E}\ \mathrm{dy} \qquad (3)$$

$$dL = \frac{F}{EA} dy \qquad (4)$$

The cross sectional area of the shaded element is:

$$A = \pi * \mathrm{x}^2 = \pi * (\frac{\mathrm{D}}{2} + \mathrm{y}\ \tan\alpha)^2 \qquad (5)$$

$$dL = \frac{F}{E\pi * (\frac{\mathrm{D}}{2} + \mathrm{y}\ \tan\alpha)^2} dy \qquad (6)$$

$$\Delta L = \int_0^t \frac{F}{E\pi * (\frac{\mathrm{D}}{2} + \mathrm{y}\ \tan\alpha)^2} dy \qquad (7)$$

$$\Delta L = \frac{Ft}{E\pi * \frac{\mathrm{D}}{2} (\frac{\mathrm{D}}{2} + \mathrm{t}\ \tan\alpha)} \qquad (8)$$

$$k_{cone} = \frac{\mathrm{F}}{\Delta \mathrm{L}} = \frac{\mathrm{E}\pi}{4\mathrm{t}} (\mathrm{D}^2 + 2\ \mathrm{t}\ \mathrm{D}\ \tan\alpha) \qquad (9)$$

C- Stiffness Of The Member (One Side Only)

$$K_{member} = K_{\mathrm{cone}} - \mathrm{K}_{\mathrm{drilled}} \qquad (10)$$

$$k_{member} = \frac{\mathrm{E}\pi}{4\mathrm{t}} (\mathrm{D}^2 + 2\ \mathrm{t}\ \mathrm{D}\ \tan\alpha) - \frac{E\pi d^2}{4t} \qquad (11)$$

$$k_{member} = \frac{\mathrm{E}\pi}{4\mathrm{t}} (\mathrm{D}^2 - d^2 + 2\ \mathrm{t}\ \mathrm{D}\ \tan\alpha) \qquad (12)$$

The corresponding formula that is widely used now is **(shigley, 2006):**

$$k_{member} = \frac{0.577\ \pi\ E\ d}{\ln\left[\frac{(1.15\mathrm{t} + \mathrm{D} - \mathrm{d})\ \ (\mathrm{D} + \mathrm{d})}{(1.15\mathrm{t} + \mathrm{D} + \mathrm{d})\ \ (\mathrm{D} - \mathrm{d})}\right]} \qquad (13)$$

It is clear that the current derived formula (Equation 12) is much compact and easy than the lengthy formula (Equation 13).
This is also the case in identical couple of members, **Figure 6**, (t=L/2) and $K_{mt}=K_m/2$ (identical series springs):

$$k_{mt} = \frac{E\pi}{8}\left(\frac{5d^2}{2L} + \sqrt{3}\,d\right) \quad (14)$$

The final member stiffness equation shows compact and direct formula as compared with the current approximate and widely used formula **(shigley, 2006):**

$$k_{mt} = \frac{0.577\,\pi E d}{2\ln\left(5\dfrac{0.577L + 0.5\,d}{0.577L + 2.5\,d}\right)} \quad (15)$$

Some researchers **(shigley, 2006)** suggested programming the above formula to save time of numerical calculation.

RESULTS AND CONCLUSIONS

Figure 7 shows the results of member stiffness according to the current work Km(c) given in Equation (14) together with the widely used formula Km(15) given in Equation (15) **(shigley, 2006)**. It is clear that the two results are adjacent. Different bolt diameter and bolt grip were chosen. The graph seems to suggest the agreement of results. This is to show the validity of the new derived formula. Any small different between the two results is to point to the accuracy of the current method , Km(c), over the conventional method Km(15).This conclusion related to the fact that both methods started with the same assumptions such as conic pressure distribution but the Km(15) uses the (ln) function which is itself a mathematical approximation. So the current method has the advantages of compact and more accurate results.

When we compare the half joint formula (12) derived in the current work with Equation (13), **(Shigley, 2006)**, we find that equation (12) has the advantage of applying any half apex angle not only 30^0. This is important for joints that use apex angles other than 30^0.

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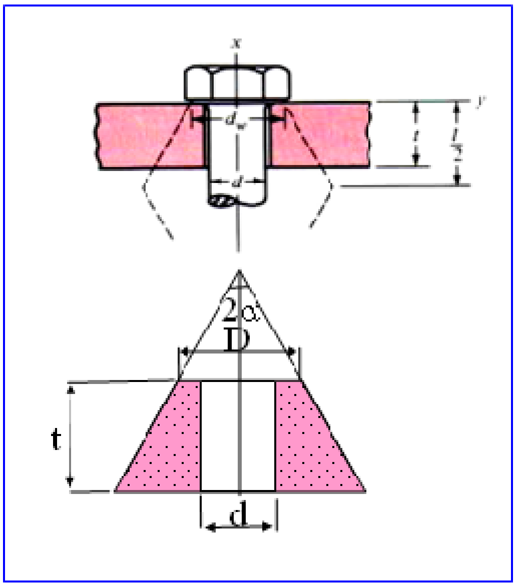

Figure 1 Clamped zone

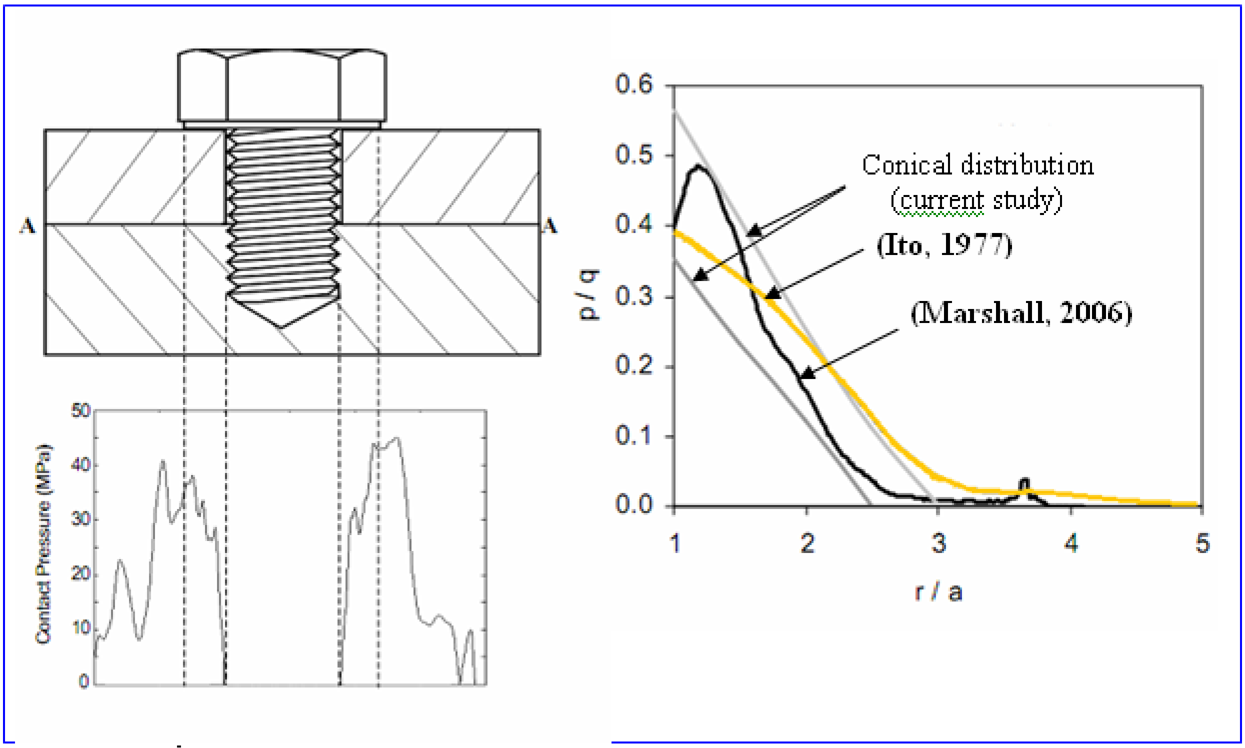

Figure 2 Approximate and actual pressure distribution **(Marshall, 2006)**

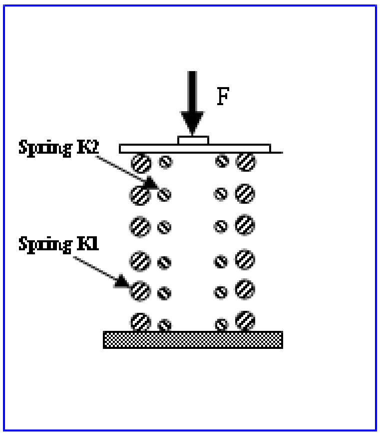

Figure 3 Kt=K1+K2

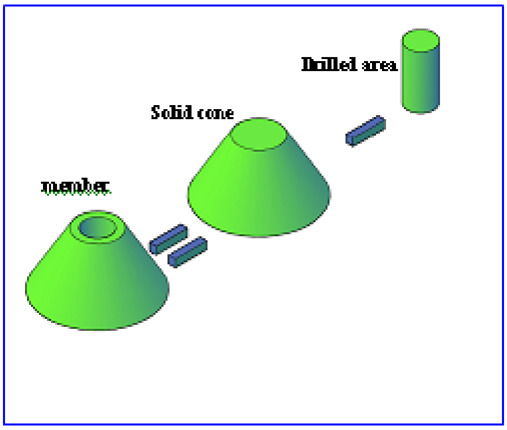

Figure 4 $K_m = K_{cone} - K_{drilled}$

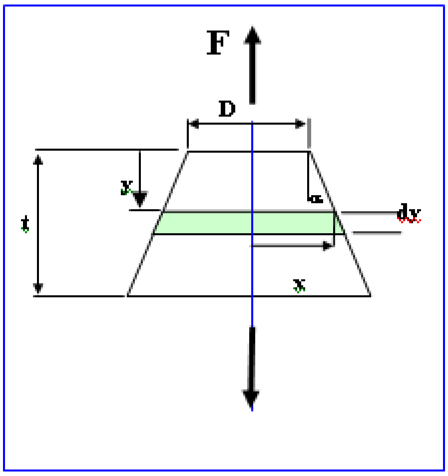

Figure 5 Stiffness derivation

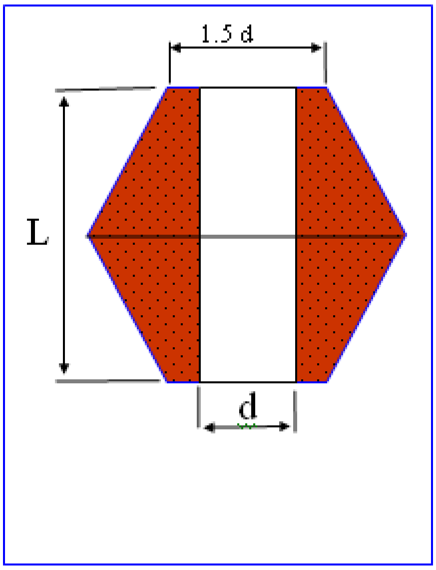

Figure 6 Identical members

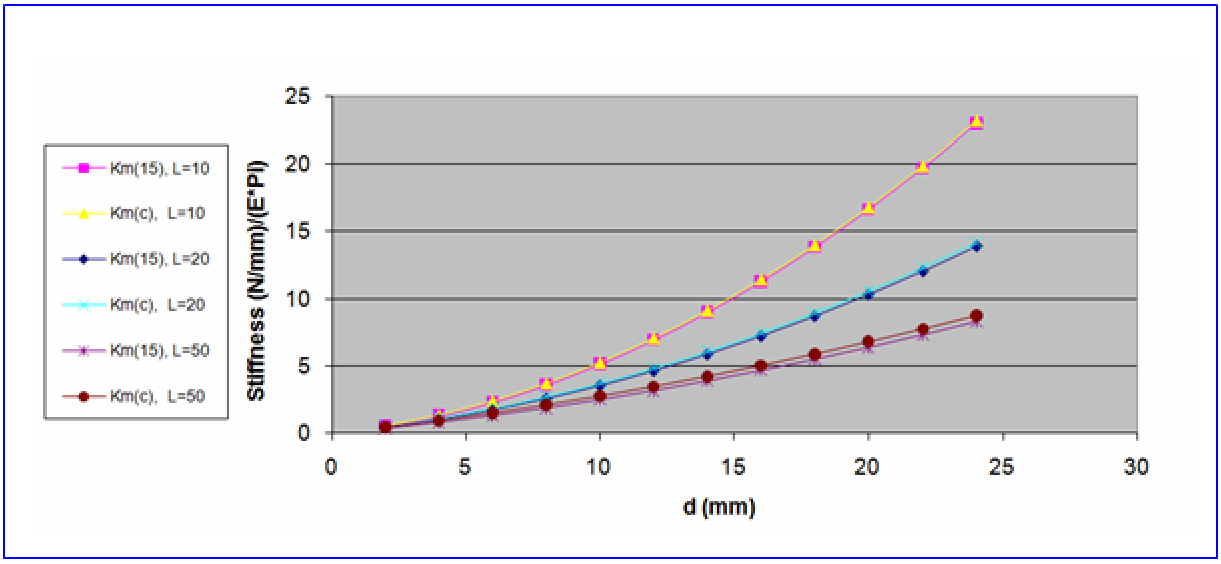

Figure 7 Current work VS (15)
Km(c) current work (14), Km (15): **(Shigley, 2006)**