

## STUDY THE EFFECT OF THE VARIATION OF LAYER'S THICKNESS ON THE BENDING CHARACTERISTICS OF THE COMPOSITE BEAM

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### ABSTRACT

The analysis bending characteristics of the composite specimen formed from steel and aluminum sheets with different thicknesses were studied here experimentally, theoretically and numerically by using finite element method.

The results show that the deflection decreases in nonlinear relationship with increase steel thickness, the maximum difference for the deflection between the experimental and finite element results was (11 %). While the moduli of elasticity and equivalent stiffness of the composite specimen increases with the increase the steel thickness.

Also the results show that the tension and compression stresses change to the bottom surface and to the top surface of the composite specimen and that depend on the position of neutral axis of the this specimen.

**KEYWORD:** Composite, beam, Bending, Stiffness, Neutral axis, Deflection

### دراسة تأثير تغيير سمك الطبقات على خصائص الانحناء للدعامة المركبة

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### الخلاصة

ان تحليل خصائص الانحناء للدعامة المركبة المؤلفة من طبقات الفولاذ والالمنيوم وبسمك مختلف قد تمت دراسته عمليا ونظريا وتحليليا باستخدام طريقة العناصر المحددة.

بينت النتائج بان الانحراف يقل بعلاقة لاطية مع زيادة سمك طبقة الفولاذ وان اقصى فرق بين النتائج العملية ونتائج العناصر المحددة كان بمقدار (11%). بينما لوحظ بان معاملات المرونة والجساءة المكافئة للعينة المركبة تزداد بزيادة سمك طبقة الفولاذ.

كذلك بينت النتائج بان تغيير اجهادات الشد والانضغاط للسطح السفلي وللسطح العلوي للعينة يعتمد على موقع محور التناظر للدعامة المركبة.

## NOMENCLATURE

Symbols	Definition	Unit
$b$	Width of the composite beam.	(m)
$E_i$	Modulus of elasticity of each constituent (i).	(N/m <sup>2</sup> )
$E_{st}$ and $E_{Al}$	Modulus of elasticity of steel and aluminum respectively.	(N/m <sup>2</sup> )
$E.I$	Equivalent stiffness.	(N.m <sup>2</sup> )
$h_o, h_1, \& h_2$	Height of layer from the bottom surface ( $h_o=0$ ).	(m)
$L$	Length of the beam.	(m)
$M$	Bending moment.	(N.m)
$P$	Applied load at the midpoint of the beam.	(N)
$R$	Radius of curvature.	(m)
$V_i$	Volume fraction of each constituent (i).	%
$V_{St}$ and $V_{Al}$	Volume fraction of steel and aluminum.	%
$\nu_{St}$ and $\nu_{Al}$	Poisson's ratio of steel and aluminum respectively.	-

## INTRODUCTION

Bending analysis of the laminated composite beam depend on the position of neutral axis, thickness of layers which effect on the central deflection of the three point bending test, equivalent stiffness, modulus of elasticity and tension and compression stresses.

One of the most important advantages obtained of the advanced composite materials lies in the possibility of getting composite materials of a suitable laminated structure of plates by arranging them with sheets. Laminated composite material consists of multi layers of various materials [1].

Therefore many studies have been accomplished to obtain composite materials suitable for practical applications.

**Bareisis and Kleiza, 2009** suggested a mathematical model for calculating the bending stiffness and fields of normal stresses at any point in the cross section of a multilayer beam. It is found from the example of two-layer beams that the normal stresses in multilayer beams under a symmetric bending considerably depend on the location of the flexural center, neutral plane, and bending stiffness relative to the principal axes of cross sections of the beams [2].

**Yaghoobi and Fereidoon, 2010** investigated the bending analysis of functionally graded simply supported beam subjected to a uniformly distribution load and found that the position of neutral surface position and the central deflection depend on the material properties of the beam vary continuously in the thickness direction [3].

**Simon etal, 2007** proposed a mathematical model and derived its analytical solution for the analysis of the geometrically and materially linear two-layer beams with different material and geometric characteristics of an individual layer by taking into account the effect of the transverse shear deformation on displacements in each layer [4].

Bareisis 2006 obtained equations allow establishing the positions of neutral layers and of the geometric, stiffness centers, stiffness for bending and shear, and calculating normal and shear stresses which depend on the mechanical properties of layers, number of layers, and their arrangement and dimensions [5].

**Jingfeng and Guoqiang, 2008** measured the stresses and deflections in various semi-continuous beams and the results show that the semi-continous composite beam are more economic and effective than the simple or continous beams. Also the semi-rigid connections affect the bending capacities and beam deflections [6].

**Ahmadreza and Luc, 2008** presented analytical model describes and improved small-deflection for piezoelectric bending beam actuators and energy conversion mechanism. This model provides an improved approach to design and analyze the performance of piezoelectric actuators [7].

**Garuckas and Bareisis, 2003** investigated the bending stiffness and strength of multilayer structural elements in relation to the mechanical properties of layers and their number layout and

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sizes. It is found that the values of normal stresses in the layers of a multilayer beam in bending on its bending stiffness and position of layers relative to the neutral axis [8].

The composite beam used in this research was made from two layers of isotropic materials (steel and aluminum) with different thicknesses.

The aim of this work is to study the variation of layer's thickness on the bending analysis of laminated composite beam which represented by the position of neutral axis, central deflection, equivalent stiffness, moduli of elasticity in both direction( longitudinal and perpendicular ) and tension and compression stresses using experimental, theoretical and finite element analysis by using ANSYS package. Also make a comparison between some results of experimental and finite element analysis.

### THEORETICAL ANALYSIS

Bending analysis of the composite beam depend on their constituents of layers, thickness of layers and geometry.

The position of neutral axis for **Figure 1** can be calculated from the following equation (1) [9]:

$$y_0 = \frac{1}{2} \cdot \frac{E_{st} \cdot (h_1^2 - h_0^2) + E_{Al} \cdot (h_2^2 - h_1^2)}{E_{st} \cdot (h_1 - h_0) + E_{Al} \cdot (h_2 - h_1)} \quad (1)$$

The Beam Bending Problem may be solved in the usual fashion except the "EI" will be replaced by the function "S" (equivalent stiffness) computed from the relation [9].

$$S = MR = \frac{b}{6} \cdot \{E_{st} \cdot [2 \cdot (h_1^3 - h_0^3) - 3y_0(h_1^2 - h_0^2)] + E_{Al} \cdot [2 \cdot (h_2^3 - h_1^3) - 3y_0(h_2^2 - h_1^2)]\} \quad (2)$$

$$S = M \cdot R = E \cdot I$$

From the values of equivalent stiffness and simple bending beam theory so as determine the theoretical value of central deflection and stresses [10]:

$$\delta = \frac{P \cdot L^3}{48 \cdot E \cdot I} \quad (3)$$

And from Simple bending theory of the beam

$$\frac{\sigma}{Y} = \frac{M}{I} = \frac{E}{R} \quad (4)$$

The following equation was used to calculate the tension and compression stresses on the bottom and upper surfaces of the beam.

$$\sigma = \frac{M \cdot Y}{I} \quad (5)$$

The rule of mixtures was used here to predict the elastic constants of the composite material. One of them modulus of elasticity in the longitudinal direction of the beam ( $E_1$ ) can be calculated by the following formula:

$$E_1 = \sum_{i=1}^n E_i \cdot V_i = E_{St} \cdot V_{St} + E_{Al} \cdot V_{Al} \quad (6)$$

The second one of modulus of elasticity in the perpendicular direction ( $E_2 = E_3$ ) can be calculated from the following formula [9]:

$$\frac{1}{E_2} = \frac{1}{E_3} = \sum_{i=1}^n \frac{V_i}{E_i} = \frac{V_{St}}{E_{St}} + \frac{V_{Al}}{E_{Al}} \quad (7)$$

Also the poisson's ratio can be calculated from the following formula:

$$\nu_{12} = \nu_{St} \cdot V_{St} + \nu_{Al} \cdot V_{Al} \quad (8)$$

Bending analyses of the composite beam behave in a different manner from that of one-element material. This difference may be viewed by deflection value and it may be higher or lower than that of the one element material according to the rule of mixture.

## EXPERIMENTAL WORK

The composite specimens were consists of two layers of different metals which represented by (steel and aluminum), some properties of these materials represented by **Table 1**. These layers were cut from plates of steel and aluminum which has a different thickness in order to obtain the total thickness of the composite beam equal to (3 mm). The two layers of steel and aluminum were bonded by using epoxy adhesive, the bottom one is steel and the upper one is aluminum with different thicknesses. **Table 2** represents the variation of layer thickness of the composite beam.

The geometry of the test specimen has a length of (100 mm) and width of (10 mm) and a thickness of (3 mm). **Figure 2** represents the three-point test apparatus with a test specimen of the bimetal composite beam and with some accessories.

The composite beam is simply supported beam with central loading equal (0, 5, 7.5, 10, 12.5.....20 N). The deflection was measured by using dial gauge, fixed at the center of the top surface of the composite beam as shown from **Figure 2**.

The experimental work was carried out in the field to determine experimentally the central deflection of the composite test specimens by using dial gauge.

## MODELING, ELEMENT SELECTION AND MESH GENERATION

The specimens are treated as a two – dimensional problem with different layers thickness. The ANSYS package is used here for this type of analysis for the beam (three point bending test).

Two – dimensional element (solid 42) is used here to modeling of solid structure. This element is defined by four nodes having two degree of freedom at each node translations in the nodal X and Y directions. The geometry, node locations, and coordinate system for this element are shown in **Figure 3** [11].

While for meshing the structure of the composite beam, it is necessary to discretize it into a sufficient number of elements. The mesh generation of this composite beam is shown in **Figure 4**.

## **RESULTS AND DISCUSSION**

**Figures 5 & 6** represent the sample of deflection contours and stress contours of the composite beam specimen with length (100 mm), width (10 mm) and thickness (3 mm divided by 1.5 mm thickness to steel layer and 1.5 mm thickness to aluminum layer), at central load ( $p=10$  N).

**Figure 7(a-e)** represents load-deflection curves for experimental and finite element analysis. It is clear that the deflection is increased approximately in linear relationship with increasing load at different rate. And the results of experimental work were higher than that of finite element analysis. The cause of difference between experimental and finite element results is due to the different between the experimental material properties of steel and aluminum that determine the experimental data of deflection and theoretical material properties that used to determine the numerical data of deflection. Where the experimental results was higher than that of finite element analysis and the maximum difference was (11 %) at aluminum layer thickness =3mm and at central load ( $P=20$  N) see **Figure 7 (a)**.

**Figure 8** shows the relationship between the deflection and the layer thickness for experimental and finite element analysis at load (12.5 N). It is seen from these figures that the deflection decrease in nonlinear relationship with increase of steel thickness and vice versa the deflection increase with increase the thickness of aluminum layer that due to different in stiffness of these materials for both experimental and finite element method [10 & 11].

**Figure 9** shows the relationship between the tension and compression stress with the layer thickness. It is clear from this figure the tension stress reach the maximum value ( $\sigma_t=16.357$  MPa.) at layer thickness (0.75 mm steel + 2.25 mm aluminum) and then the tension stress decrease with increase the thickness of steel layer. Also it can be seen that the compression stress reach the minimum value ( $\sigma_c=9.042$  MPa.) at layer thickness (1.5 mm steel + 1.5 mm aluminum) and then the compression stress increases with increase the thickness of steel layer that due to the change in the position of neutral axis of laminated composite beam and that coming from inhomogeneous of the composite beam due to changing the thickness of each layer [5 & 8].

**Figure 10** shows the relationship between the position of neutral axis and the thickness of steel and aluminum layers. It is clear from this figure that the position of neutral axis decrease with increase thickness of steel layer and reach minimum value at ( 1.5 mm steel + 1.5 mm aluminum) and then increase with increase the layer thickness of steel layer that due to the changing in the thickness of each layer [3].

**Figure 11** shows the relationship between the equivalent stiffness and thickness of steel and aluminum layers. It is clear from this figure that the equivalent stiffness increase with the increase the thickness of steel layer and vice versa decrease with increase the thickness of aluminum layer in nonlinear relationship that due to the stiffness of each materials (i.e. tensile properties of steel higher than tensile properties of aluminum) in additional to increase the thickness of steel layer, because the equivalent stiffness equal to (EI) and this is consistent with the results of reference [2 & 8].

**Figure 12** shows the relationship between the modulus of elasticity in parallel and perpendicular direction to the composite beam with the thickness of the steel and aluminum layers. It is clear from this figure that the modulus of elasticity in parallel direction and modulus of elasticity in perpendicular direction increase linearly and nonlinearly relationship respectively with the increase the thickness of steel layer according to the rule of mixtures [1].

## **CONCLUSIONS**

The following points were concluded from this result:

- 1-) The deflection decreases in nonlinear relationship with the increase the steel thickness for both experimentally and finite element results.
- 2-) The maximum difference for the deflection between the experimental and finite element results was (11 %).

- 3-) The tension and compression stresses change from bottom surface to top surface of the composite specimen which depend on the position of neutral axis.
- 4-) Equivalent stiffness and moduli of elasticity in both directions increases with increase the steel thickness.

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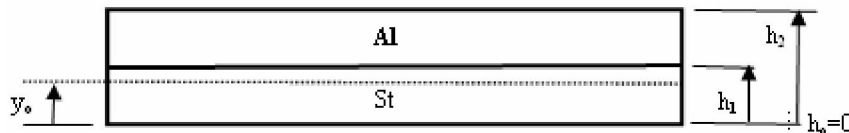
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**Table 1** Properties of materials used [1].

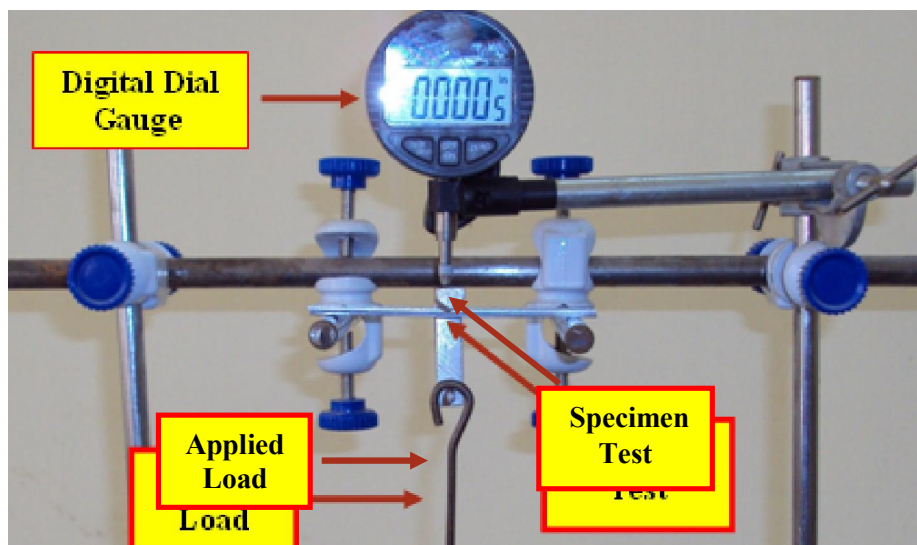
	Density g/cm <sup>3</sup>	Modulus of elasticity (GPa.)	Poisson's ratio	Tensile strength (MPa.)	Percentage Elongation (%)
Steel	7.8	200	0.3	415	20
Aluminum	2.71	69	0.33	90	40

**Table 2** Variations of layer thickness and some properties of composite beam.

	Composite Beam				
	3	2.25	1.5	0.75	0
Steel layer thickness (mm)	3	2.25	1.5	0.75	0
Aluminum layer thickness (mm)	0	0.75	1.5	2.25	3
Total Thickness (mm)	3	3	3	3	3
Modulus of elasticity in the longitudinal direction (GPa.)	200	167.25	134.5	101.75	69
Modulus of elasticity in the perpendicular direction (GPa.)	200	135.63	102.6	82.51	69
Poisson's ratio in plane 1-2	0.3	0.3075	0.315	0.3225	0.33
Volume fraction of steel %	100	75	50	25	0
Volume fraction of Aluminum %	0	25	50	75	100



**Figure 1** Composite Beam.



**Figure 2** Bending Apparatus with Specimen Test.

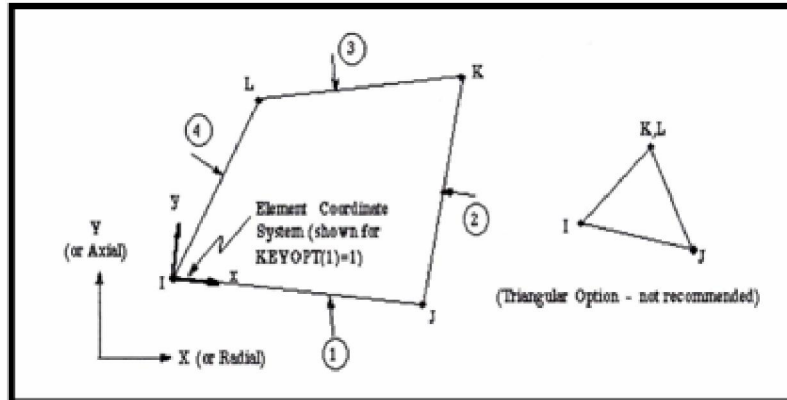


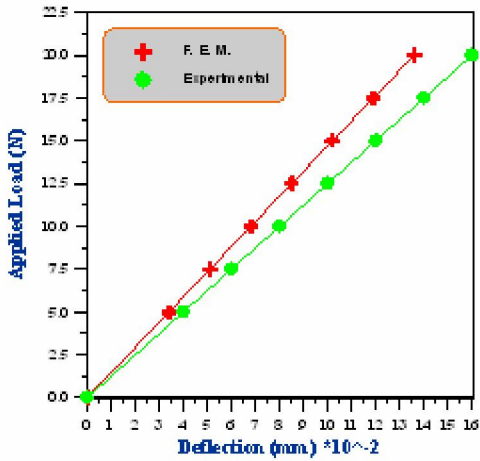
Figure 3 Two-Dimensional element (Solid-42) [12].



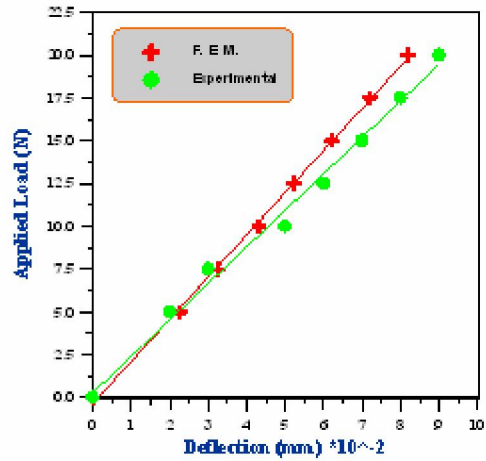
Figure 4 Mesh Generation of the Composite Beam.



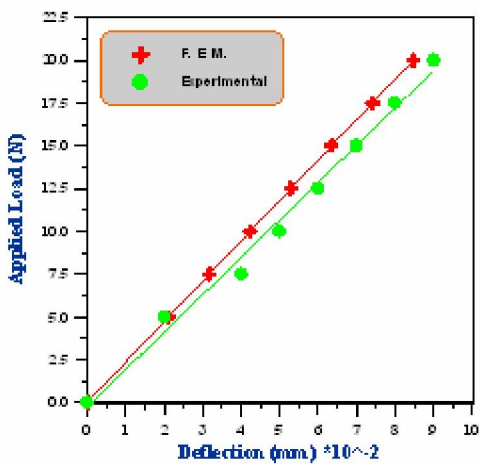
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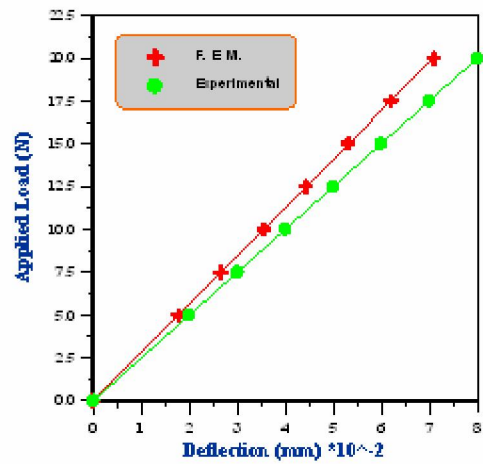
(a) 3mm AL



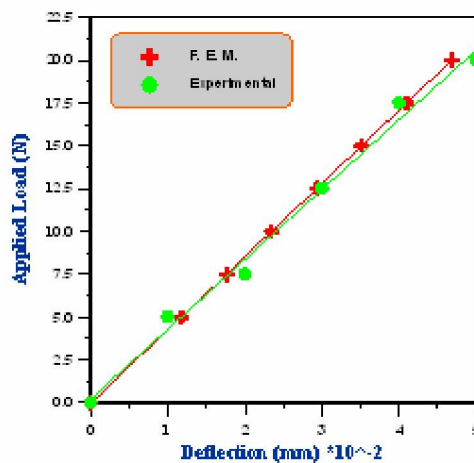
(b) 0.75 mm St & 2.25 mm AL



(c) 1.5 mm St & 1.5 mm AL

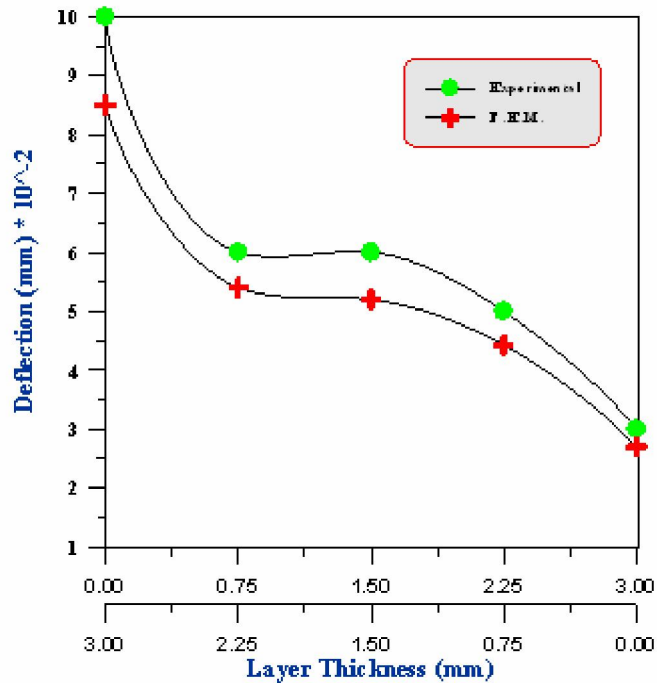


(d) 2.25 mm St & 0.75 mm AL

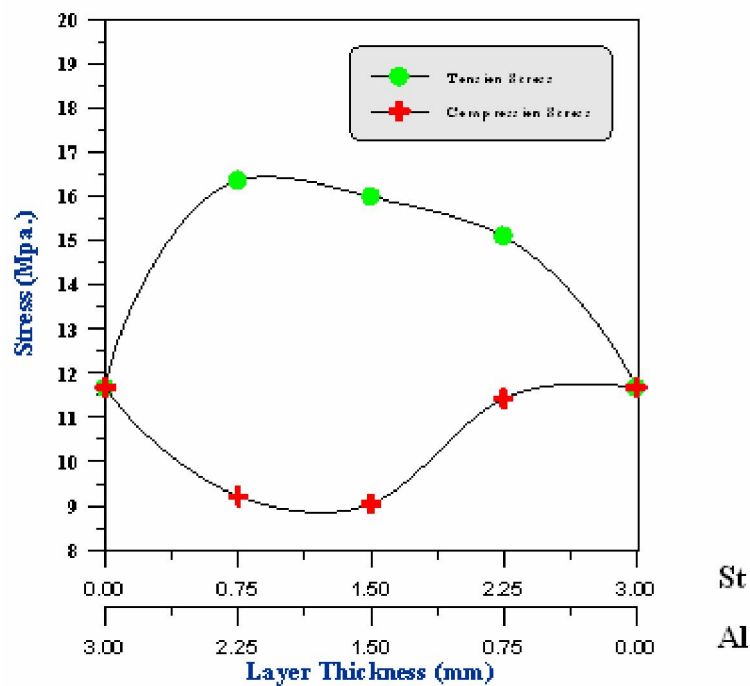


(e) 3mm St

**Figure 5** Relationships Between Deflection and Load for Experimental and Finite Element Method for the Beam with Length (100 mm).

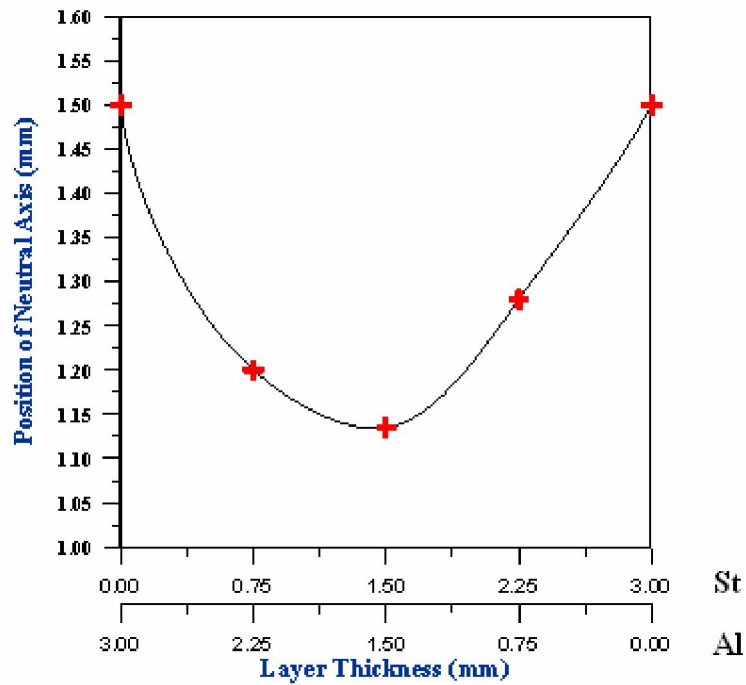


**Figure 6** Relationship Between Deflection and Layer Thickness of Laminated Composite Beam for Experimental and Finite Element Method for the Beam with Length (100 mm) and central Load (12.5 N) (equation3).

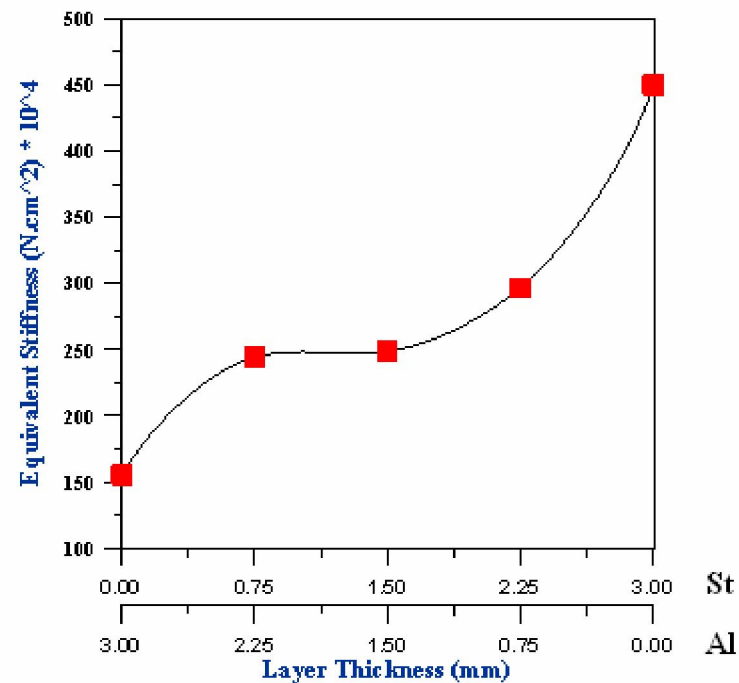


**Figure 7** Relationship Between Stress and Layer Thickness for Laminated Composite Beam for the Beam with Length (100 mm) and Load (10 N) (equation5).

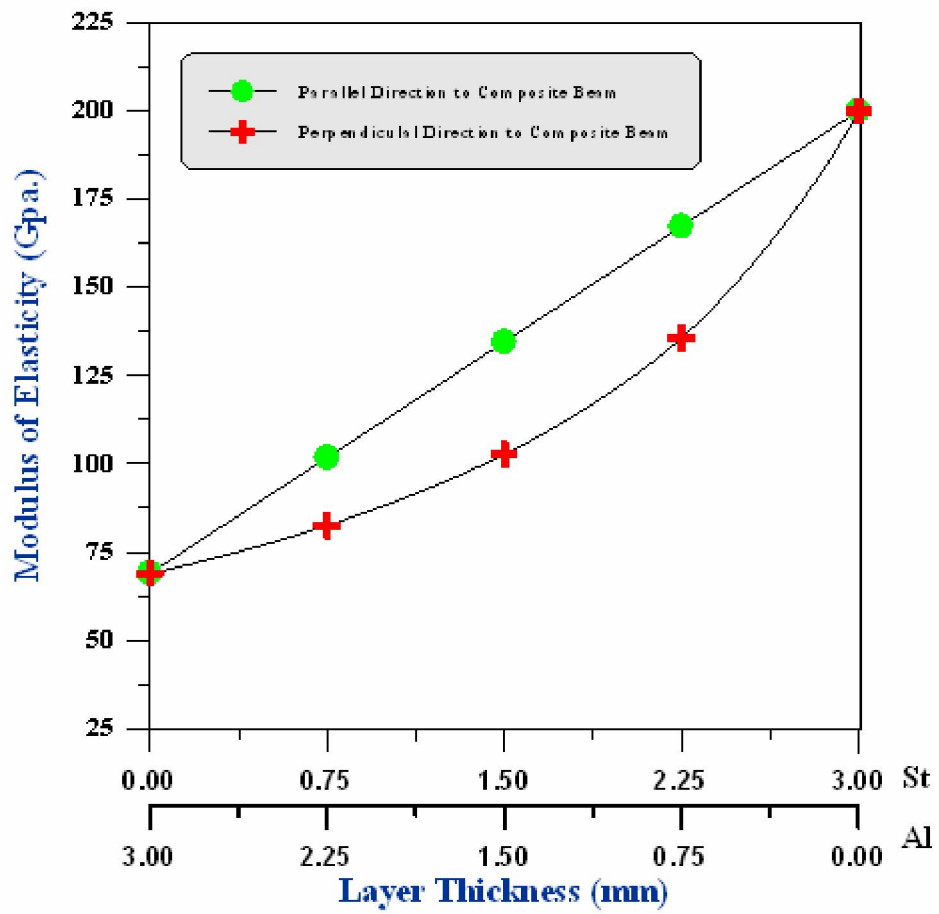
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**Figure 8** Relationship between Position of Neutral Axis and Layer Thickness for Laminated Composite Beam (equation 1).

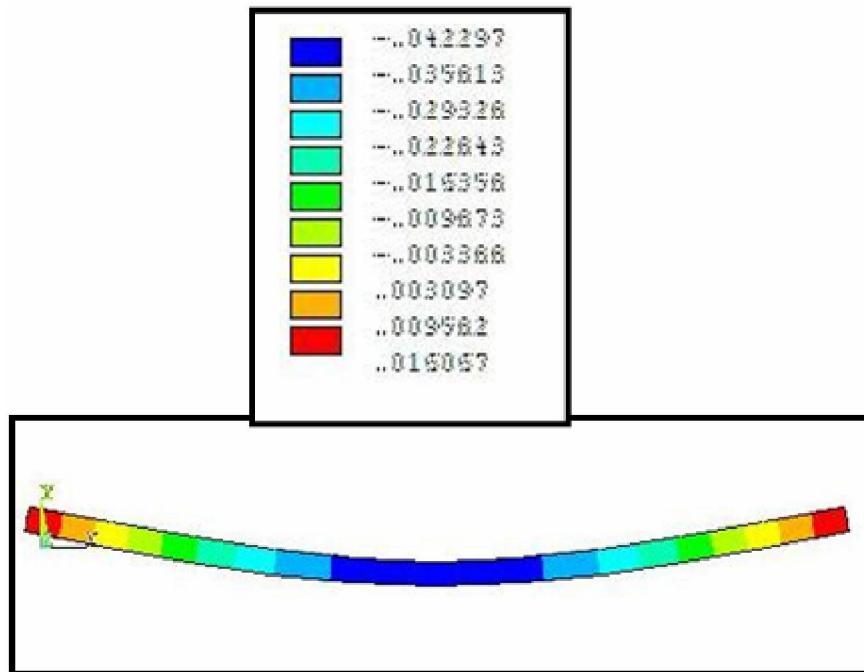


**Figure 9** Relationship between Equivalent Stiffness and Layer Thickness for Laminated Composite Beam (equation2).

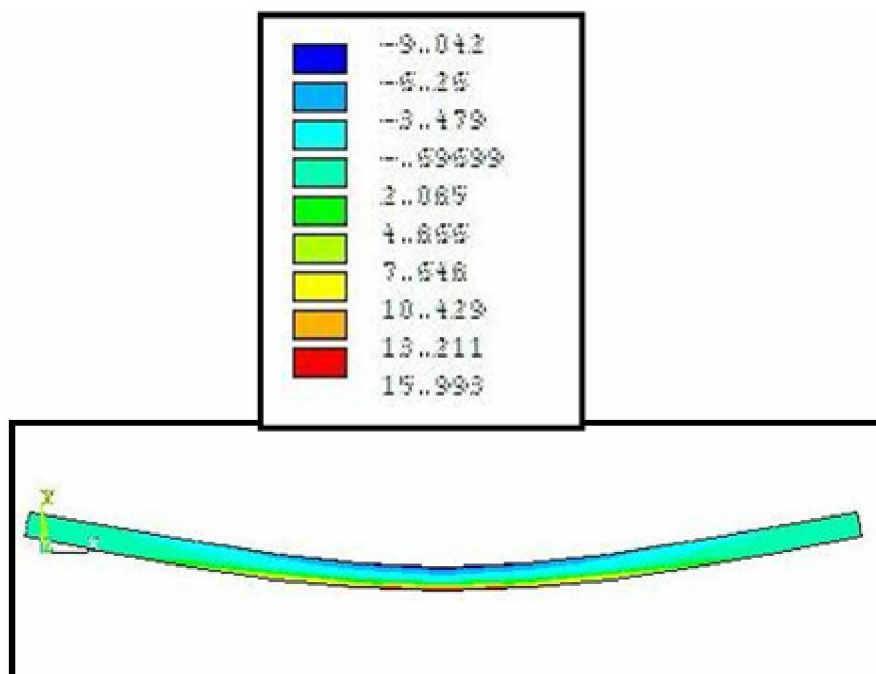


**Figure 10** Relationship between Modulus of Elasticity and Layer Thickness for Laminated Composite Beam (equation 6 and 7).

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**Figure 11** Sample of Deflection contours of Laminated Composite Beam for 1.5 mm Steel and 1.5 mm Aluminum and at central load =10 N.



**Figure 12** Sample of Stress Contours of Laminated Composite Beam for 1.5 mm Steel and 1.5 mm Aluminum and at central load =10 N.