CONTROL OF ASYMMETRIC FLOWS BY APPLIED MAGNETIC FIELD IN A SYMMETRIC SUDDEN EXPANDED CHANNEL

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Abstract

Investigate the symmetry-breaking flow bifurcation phenomena and its control of an electrically conducting generalized Newtonian liquids flowing through symmetric sudden expansion channel by applied magnetic field in the transverse direction studied in this paper. The governing nonlinear magnetohydrodynamic equations simplified for low conducting liquid metals are written and solved numerically using PISO-GNFMHD algorithm, which was developed by the author to include generalized Newtonian fluids, as well as the effects of the magnetic field on the fluid flow based on the finite volume method. The effects of non-Newtonian rheology and the magnetic force on flow bifurcation and separation are studied. In case of power-law fluid model the flow transition depends nonlinearly on shear viscosity in particular the shear-thickening fluids delay the onset of bifurcation where as shear-thinning fluids the early flow transition is occurred. The magnetic force is always delay this transition. The main conclusion is the possibility to control the phenomenon of bifurcation finally by applying an external magnetic field.

Keywords: Bifurcation, Asymmetric flow, Rheology, Magnetic field, Sudden expansion, Finite volume method.

التحكم بالجريان غير المتماثل بواسطة تسليط مجال مغناطيسي في القنوات المتماثلة ذات التوسع المفاجئ

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الخلاصة:

فحص ظاهرة التشعب التي تحدث خلال جريان السوائل الغير النيوتونية في القنوات المتناظرة ذات التوسع المفاجئ والسيطرة عليها بواسطة تسليط مجال مغناطيسي مستعرض درست في هذا البحث. المعادلات غير الخطية الهايدرومغناطيسية للسوائل كتبت وحلت عدديا بواسطة لوغارتيم PISO-GNFMHD الذي طور من قبل المؤلف ليشمل السوائل الغير النيوتونية بالاضافة الى تاثير المجال المغناطيسي على جريان المائع على اساس طريقة الفروق الحجمية. تمت دراسة تاثيرات الريولوجيا غير النيوتونية والقوة المغناطيسية على ظاهرة التشعب والانفصال. في حالة موديل السائل (power-law) فانتقال الجريان يعتمد لا خطيا على لزوجة القص وعلى وجه الخصوص في سوائل القص-السميك يتأخر في ظهور التشعب بينما في حين سوائل القص-الرقيق يحدث الانتقال في وقت مبكر. القوة المغناطيسية دائما تؤخر هذا الانتقال. ومن الجدير أن نلاحظ أن ظاهرة تدفق التشعب يمكن السيطرة عليها تماما باستخدام قيمة مناسبة لمجال مغناطيسي يسلط خارجيا.

Notation

x,y	axial and radial coordinates	τ_{xy}	shear stress
u	axial velocity	K	consistency coefficient
v	radial velocity	N	power-law index
ρ	density of the fluid	η	fluid viscosity
h	upstream channel height	B_0	magnetic field
H	downstream channel height	$\mathrm{Re} = v_0\, L/\eta$	Reynolds number
E	expansion ratio E=H/h	$\mathrm{Re}_m = \mu\sigma\, L\, v_0$	magnetic Reynolds number
u	non-dimensional velocity vector	$Ha = LB_0\sqrt{\sigma/\rho\eta}$	Hartmann number
p	kinetic pressure scaled	$N = Ha^2/\mathrm{Re}$	interaction parameter
L	characteristic length	$N(J \times B)$	Lorentz force
J	current density		
σ	electrical conductivity		
μ	magnetic permeability of the fluid and the walls		

1. Introduction

There have been numerous attempts over the past four decades to understand the flow bifurcation phenomena in a symmetric expansion planar geometry, both from experiment and numerical simulation. At low Reynolds numbers the fluid flow in a symmetric sudden expansion was found to separate symmetrically with equal sized recirculation attached to the upper and lower walls as expected. As Reynolds number was increased the separation regions increased in size and upon exceeding the critical Reynolds number for symmetry-breaking bifurcation the flow became unstable with an instability manifested as an asymmetric separation of the flow. As the Reynolds number increases further, the flow becomes oscillatory and then finally turbulent.

The present work concentrates the flow characteristics of expansion channel. Expansions have important applications in engineering processes like in flows around building, free jets, refrigeration, heat exchangers, ducts for industrial use, and extrusion. This is one of the simplest geometries for flow analysis with potential applications in engineering and extent in biological circulatory systems. The multiple re-circulating vortices and their counter-rotating characters have fundamentally interest that attracted many researchers.

The studies of Newtonian fluid flows in planar sudden expansion of various ratios and conditions are now benchmark problem which have been analyzed by many workers, [1]-[4]. Much information about bifurcation Phenomena is known, like the critical Reynolds number for asymmetric flow structures and bifurcation phenomena. For non-Newtonian flows, such investigation is recent and there isn’t much information about it. A better understanding of non-Newtonian flow through sudden expansion should lead to both the design and development of hydrodynamically more efficient processes leading to an improved quality control of the final products [5]-[7]. In spite of an exhaustive studies of generalized Newtonian fluid flows through sudden symmetric expansion no results regarding the flows of an electrically conducting generalized Newtonian fluid has been reported so far.

This is now a benchmark flow problem. On the other hand, flow separation has a considerable impact on the flow structure as a whole and has been the subject of intensive research works for many years. With views of these phenomena, the controlling of flow field is an immense important from engineering point of view. There are several methods to control on the asymmetric flows such as (i) shape of geometry[8], (ii) suction/blowing through porous boundary[9], (iii) put a solid block in the stream flow or on the walls of the channel [8] (iv) the use of external magnetic fields and applied on the fluid flow [10]-[11], (v) viscoelastic force and many others. Asymmetric flows of viscoelastic fluids in planar sudden expansion channel had been extensively analyzed by

Gerardo N. R. et al. [12]. Some studies were aimed to increase hydrodynamic efficiency by controlling the flow, including the alteration of structure of the flow [13]-[15] .

The main objectives of the present study are: (i) to examine possibility the control of Bifurcation Phenomena for non-Newtonian fluid flow in the presence of a transverse magnetic field. (ii) to investigate the effects power law index (n) for non-Newtonian fluids on the Bifurcation Phenomena in the planar sudden expansion in the presence or without presence an externally applied magnetic field.

2. Numerical simulation of magnetohydrodynamic equations

In the present work we consider the two-dimensional incompressible, isothermal flow from a straight channel of height h to the width of the expanded channel H, corresponding to an expansion ratio E, (E=H/h=3), applied on the fluid flow by an external magnetic field, as shown in Fig.1.

Under the above assumptions the equations governing the flow under consideration are [10], [16]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{………} (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}-\left(\frac{\sigma B_0^2}{\rho}\right)u \qquad \text{………} (2)$$

where u and v are the the axial and the radial velocity components in the x-axes and y-axes, respectively. ρ is the density of the fluid, τ_{xy} is the shear stress, B_0 is the strength of the magnetic field and σ is the electric conductivity.

The shear stress tenser is defined by power law model [17]

$$\tau_{ij}=2K\left(2D_{kl}D_{kl}\right)^{(n-1)/2}Dij \qquad \text{……… (3)}$$

where

$$Dij=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \qquad \text{……… (4)}$$

where the *n* is the power-law index and K is the consistency coefficient.

The dimensionless mass conservative and Navier–Stokes equations are:

$$\nabla.u=0 \qquad \text{……… (5)}$$

$$u.\nabla u=-\nabla p+\frac{1}{\text{Re}}\nabla^2 u+N(J\times B) \qquad \text{………} (6)$$

where u is the velocity vector (u=($u.v$)) and p is thel kinetic pressure scaled with v_0 and $\rho v_0^2=0$ respectively.

Other parameters can be defined as the Reynolds number is the $\text{Re}=v_0\,L/\eta$; the magnetic Reynolds number is the $\text{Re}_m=\mu\sigma L v_0$; the interaction parameter is the $N=Ha^2/\text{Re}$; the Magnetic parameter (Hartmann number) is the $Ha=LB_0\sqrt{\sigma/\rho\eta}$ and the Lorentz force is the $N(J\times B)$.

For low magnetic Reynolds numbers, the fully developed MHD flow in a rectangular channel, where flow velocity is in the x-direction and the external magnetic field is applied in the y-direction.

Ohm's law for the density of induced electric currents can be written as:

$$J=-\nabla\varphi+u\times B \qquad \text{……… (7)}$$

The electric current is conservative:

$$\nabla . J = 0 \quad \ldots\ldots\ldots (8)$$

Combining equation (7) and equation (8), we get the electrical potential equation:

$$\nabla . (\nabla \varphi) = \nabla . (u \times B) \quad \ldots\ldots\ldots (9)$$

A set of non-linear equations above are solved numerically using PISO-GNFMHD algorithm (PISO algorithm for Generalized Newtonian Fluid and Magnetohydrodynamics (MHD)), which was developed by the author to include generalized Newtonian fluids, as well as the effects of the magnetic field on the fluid flow based on the finite volume method (PISO algorithm) [18], [19].

For incompressible MHD flow, A non-staggered grid arrangement used in our numerical scheme, offering advantages over the collocated arrangement especially in convective dominated flows. The upwind scheme is introduced to the discretized equations to overcome problems concerning high convection terms in the momentum. The upwind scheme provides a first order accuracy instead of second order that the diffusion terms retain, leading to inaccurate solution when the local velocity gradients are large. To overcome this problem the “deferred correction” approach was utilized. In this scheme, higher-order flux approximations (central difference scheme) are solved explicitly and this approximation is then jointed with an implicit low-order approximations (upwind difference scheme). The relaxation factor used for the momentum equations was equal to 0.7 whereas for the pressure correction equations it was equal to 0.3.

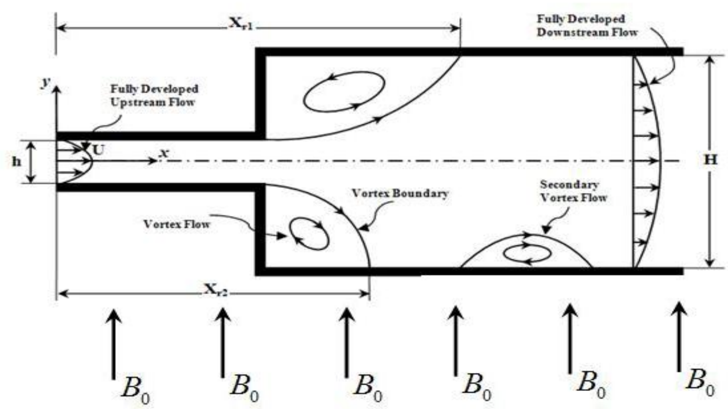


Figure (1): Schematic representation of the planar sudden expansion with magnetic field.

The boundary conditions are set according to the following:

- A fully-developed (Poiseuille axial-velocity profile) at the inlet and the outlet of the channel.
- On the walls, no-slip boundary conditions are applied:
  $u = 0 \;;\; v = 0 \quad \ldots\ldots\ldots (10)$
- At the inlet of the channel, the axial and the radial velocity components are set as:
  $$u(y) = 1.5U\left[-\left(\frac{y}{h/2}\right)^2\right] \;;\; v = 0 \quad \ldots\ldots\ldots (11)$$
- At the outlet of the channel, the axial and the radial velocity components are set as:
  $\partial u/\partial y = 0 \;;\; \partial v/\partial y = 0 \quad \ldots\ldots\ldots (12)$
- The inlet and outlet values of pressures are extrapolated depending on the inflow pressure value.
- The electric potential for walls is put according to Newmann boundary condition $\partial\varphi / \partial n$.

3. Results and Discussion:

In this paper we mainly studied the flow bifurcation and separation phenomena for symmetric planar expansion geometry for low conducting fluids in presence of externally applied magnetic field. The results are predicted based on numerical solution of incompressible nonlinear equations for non-Newtonian fluid model (power-law) under the approximations of low conducting fluids and usual MHD approximation to fluid motion. Fig.2 shows the pattern of streamlines for sudden expansion channel[8]. The figure clearly indicates that the two symmetric re-circulating bubbles are formed at the flow Reynolds number 25 but the bubbles are asymmetric shapes at Re=48 and further increase value of Re, say 80 the bubbles are completely in different shapes. Bifurcation diagrams (vortex size asymmetry) show the fluid flow transition from symmetric to nonsymmetric state in terms of the difference (Dx= (Xr1−Xr2)) between the size of a upper and lower vortex distributions along the channel walls against the Reynolds number [8], [20], [21]. When the distance (Dx) is equal to zero, the flow symmetric; but if (Dx) is equal to non-zero, the flow can be a formality two asymmetric fluid flow after the critical Reynolds number, as shown in Fig.2.

Fig.3 presents the bifurcation diagram (Dx versus Re) for three values of n, the flow consistency parameter of power-law model. The diagram indicates that the non-Newtonian rheology, that is the nonlinear shear fluid viscosity has an influence on bifurcation point. In particular, for shear thickening fluids the critical value of the onset of symmetry breaking bifurcation is decreased and increased with shear thinning fluids in compare with the Newtonian fluids.

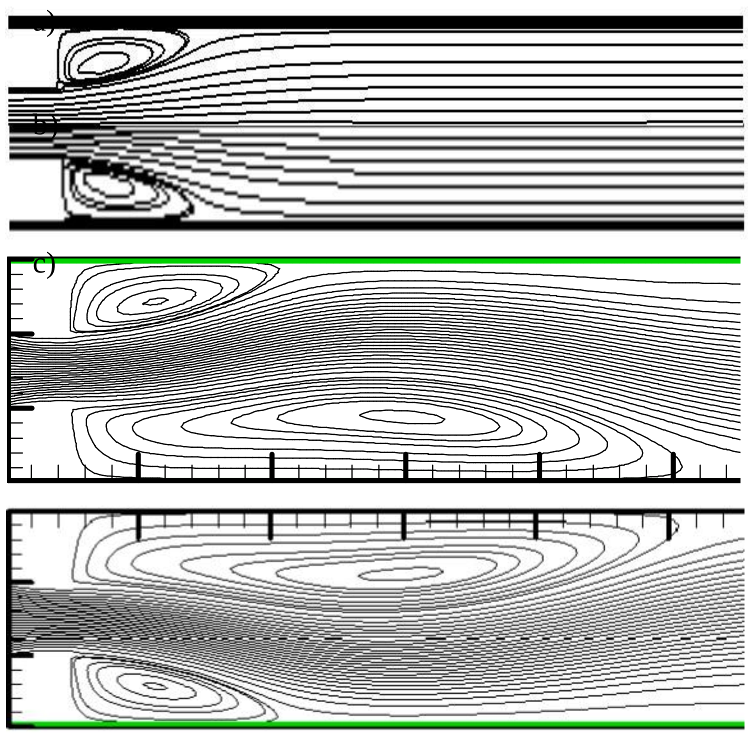


Figure (2): Streamline contours at various Reynolds numbers for Newtonian fluid flow. a) Re=25, b) Re=80 c) Re=80; [8].

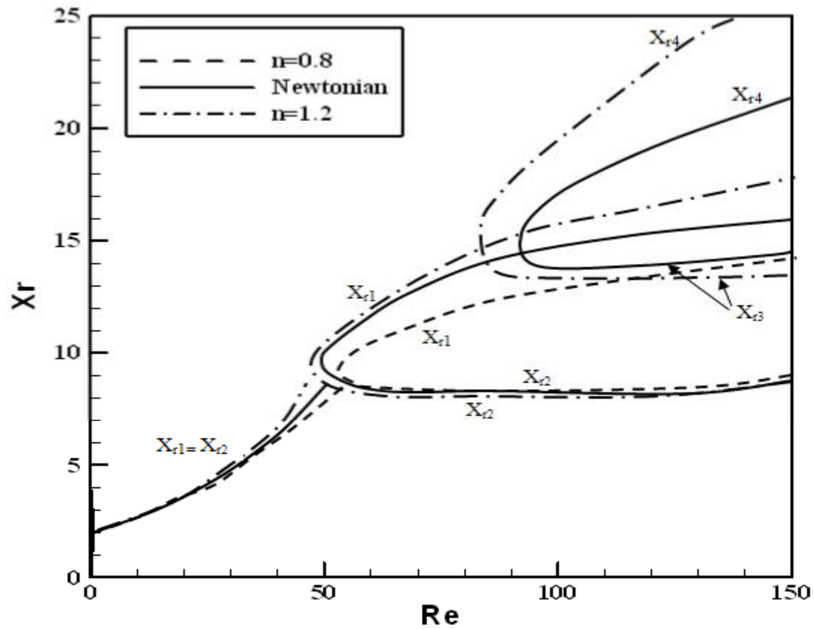


Figure (3): Bifurcation diagram with various Reynolds numbers for generalised Newtonian fluids (n = 0.8, 1 and 1.2); [8].

3.1 Nodalization analysis

To study the effect of a mesh refinement on numerical results, we used Richardson's extrapolation method and have been five computational mesh as given in Table (1) and (2). The mesh-converged value is given as [7], [22]:

$$\Phi = \frac{(N_V / N_{IV})^{OS}\Phi_V - \Phi_{IV}}{(N_V / N_{IV})^{OS} - 1} \qquad \dots\dots\dots (13)$$

and the relative error as:

$$E_r = \left| \frac{\Phi_V - \Phi}{\Phi} \right| \times 100 \qquad \dots\dots\dots (14)$$

where Φ_V and Φ_{IV} are values of particular flow variables corresponding to a Mesh V and Mesh VI. N_V and N_{IV} represents the number of elements for Mesh V and Mesh VI.

We use a mesh independence test for verification of the accuracy of the results by taking the five computational mesh 200×30, 400×60, 600×80, 800×100 and 1000×120. Tables (1) and (2) show the accuracy of data for recirculation lengths and the separation point (Xr_1) and (Xr_2) on the upper and lower walls for Re=100 and for with and without applied magnetic field (M=0, 2). Computational grid at the channel is represent as in Fig.4, which became coarser at the center of channel while close to the walls the grid was finer.

The Mesh IV (1000×120) and Mesh V (800×100) are shown in Table (1) and (2) agree well. The estimated numerical accuracy for recirculation lengths without applied magnetic field (M=0) equal to E_r=0.0308% and 0.0039% for Xr_1 and Xr_2 respectively. while the estimated numerical accuracy with applied magnetic field at M=2 equal to E_r=0.0028% and 0.0012% for Xr_1 and Xr_2 respectively. In the end of this test we chose the mesh V (1000×120) for all calculations in this work.

Table (1): Effect of mesh refinement for Newtonian fluid with M=0 and Re=100.

Mesh	Number of elements	(Xr_1)			(Xr_2)		
		Xr_1	Φ	Er[%]	Xr_2	Φ	Er[%]
Mesh I	200 × 30	12.9543	----	----	7.4868	----	----
Mesh II	400 × 60	14.6215	14.7326	0.7544	8.0998	8.14067	0.5020
Mesh III	600 × 80	14.7752	14.8264	0.3456	8.1710	8.19473	0.2896
Mesh IV	800 × 100	14.8010	14.8155	0.0980	8.1748	8.17694	0.0261
Mesh V	1000 × 120	14.8067	14.8113	0.0308	8.1752	8.17552	0.0039

Table (2): Effect of mesh refinement for Newtonian fluid with M=2 and Re=100.

Mesh	Number of elements	(Xr_1)			(Xr_2)		
		Xr_1	Φ	Er[%]	Xr_2	Φ	Er[%]
Mesh I	200 × 30	10.9443	----	----	6.2916	----	----
Mesh II	400 × 60	11.5301	11.5692	0.3376	6.4267	6.43571	0.1399
Mesh III	600 × 80	11.5402	11.5436	0.0292	6.4292	6.43003	0.0130
Mesh IV	800 × 100	11.5438	11.5458	0.0175	6.4301	6.43061	0.0079
Mesh V	1000 × 120	11.5442	11.5445	0.0028	6.4302	6.43028	0.0012

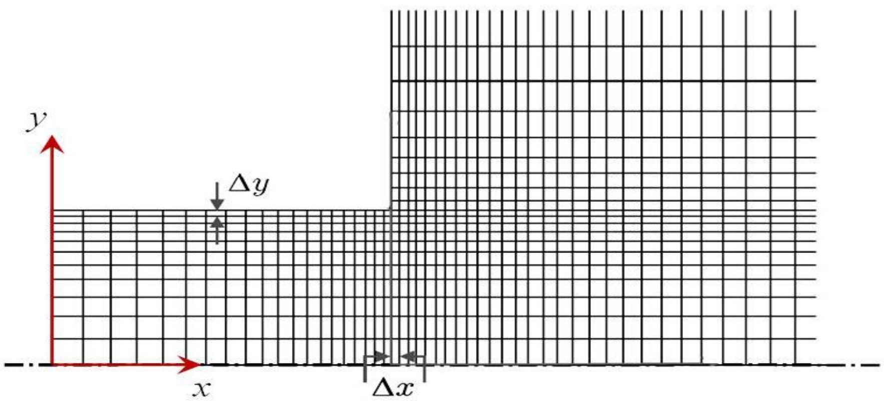


Figure (4): Computational grid at the channel.

3.2 MHD flow at low magnetic Reynolds number

Fig.(5) shows the stream function for Newtonian fluid at $\mathrm{Re} = 150$ with different values of Hartmann numbers starting from $M = 0$ to $M = 10$ and explains how the magnetic field affects the size of the recirculation.

At $M = 0$, the flow is asymmetric and the secondary vortex appears due to adverse pressure gradient. The size of the recirculation starts reducing for $M = 0.5$ and with increasing applied the magnetic field the separation area shrinks further. The flow become symmetric for $M \geq 2.5$ because the increase of Lorentz force.

Fig.(6) represents compare between the bifurcation results for Newtonian and Non-Newtonian fluids and for different values of the Hartmann number (M=0, 1and 2).

For the Newtonian fluid, the symmetric flow is maintained up to Re=47.91, as shown in Fig.(6b) and with applied magnetic field (M =1), the flow remains symmetric up to Re=75.21 and up to Re=90.05 for M=2.

For Pseudoplastic fluid (n=0.8), as shown in Fig.(6a), the symmetric flow is maintained up to Re = 51.00 and with applied magnetic field (M =1), the flow remains symmetric up to Re=80.13 while up to Re=105.26 for M=2.

For Dilatant fluid (n=1.2), in Fig.(6c), the symmetric flow is maintained up to Re=45.62 and with applied magnetic field (M =1), the flow remains symmetric up to Re=69.42 and up to Re=83.31 for M=2.

Here we can observe with an external magnetic, the value of the bifurcation Reynolds number is increased, as shown in Table (3). Also From Fig.6 we can observe that the effect of power law index (n) on the critical Reynolds number Re_{cr} where the critical Reynolds number is delayed for Pseudoplastic fluids (n <1) while forward for Dilatant fluids (n >1) relative to the Newtonian fluids (n =1).

In order to assess the accuracy of present numerical model, the present results for critical Reynolds number Re_{cr} in Newtonian fluid have been compared with numerical and experimental data obtained by previous studies as Khalaf A.H. (2012) [8], Fearn R.M., et al., (1990) [1] and Luo L., (1997) [23], and the comparison show a good agreement, as shown in Table (3).

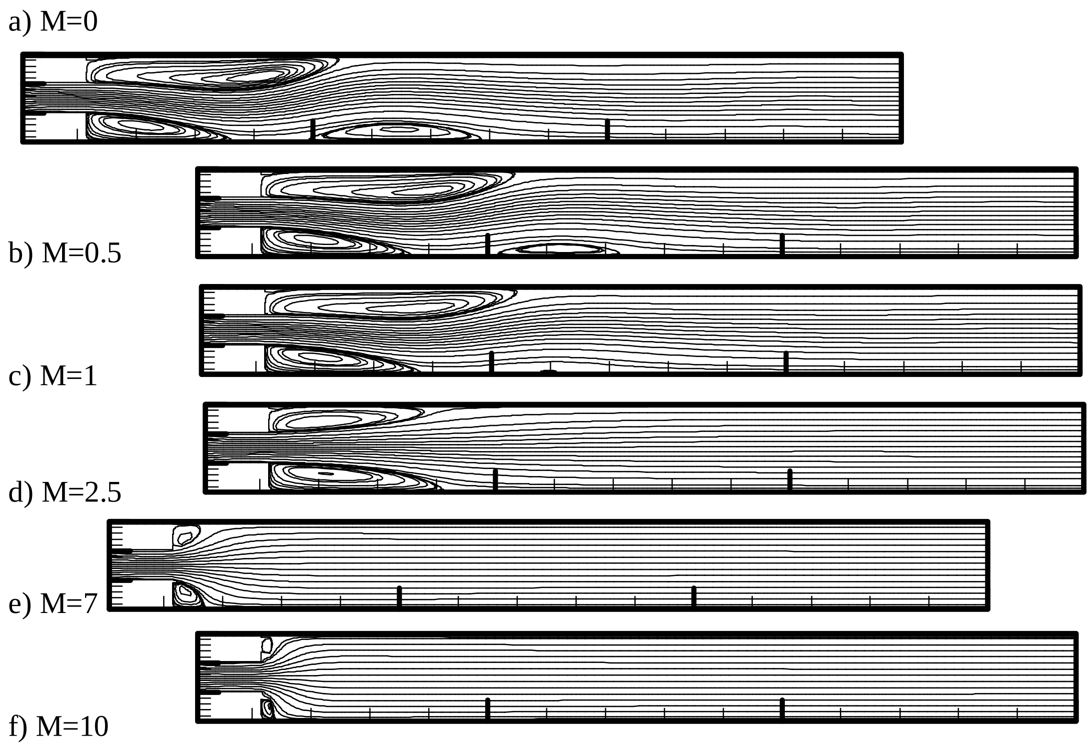


Figure (5): Stream function for Newtonian fluid with Re=150 and various Hartmann numbers (M).

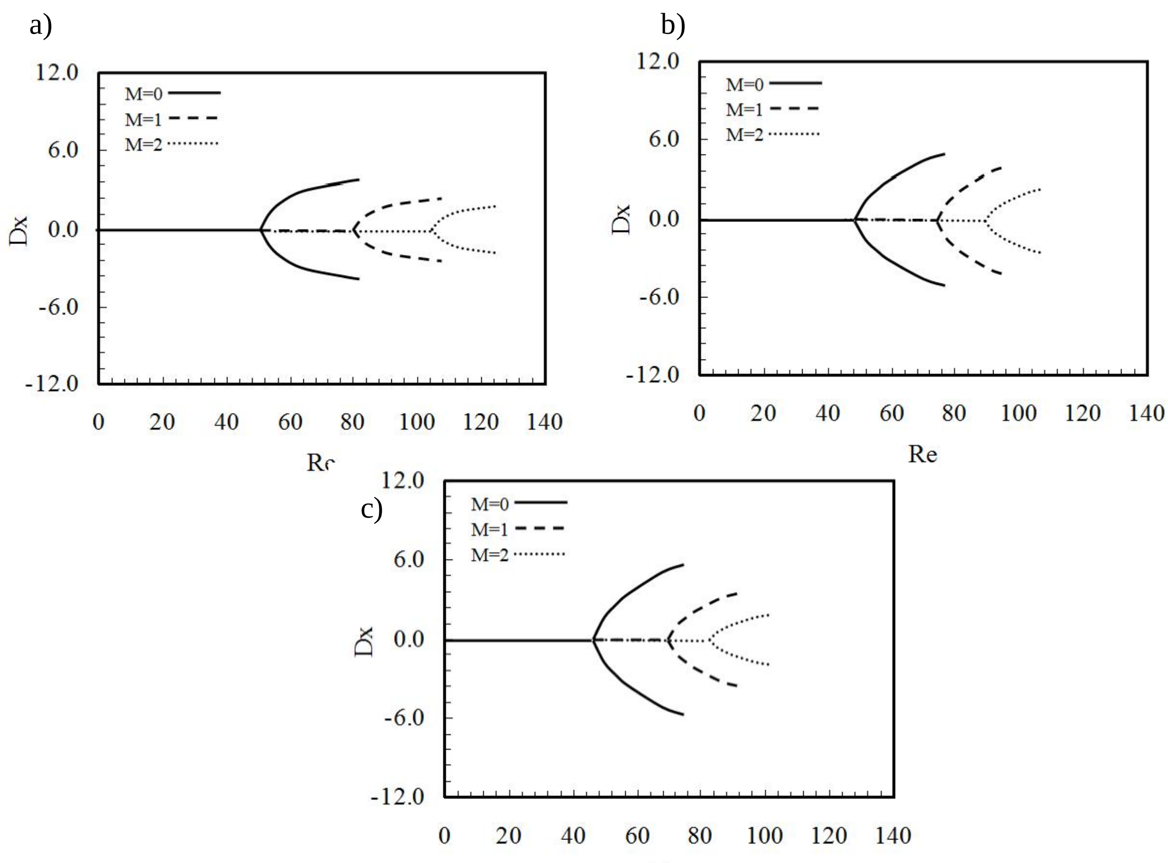


Figure (6): Bifurcation diagram for Non-Newtonian fluids for various Hartmann numbers (M) . a) n=0.8, b) n=1, c) n=1.2

Table (3): Critical Reynolds numbers for bifurcation determined for generalised Newtonian fluids for various Hartmann numbers (M).

Re_{cr}	M=0				M=1 (present work)	M=2 (present work)
	Khalaf A.H. (2012) [8]	Fearn R.M., et al., (1990) [1]	Luo L., (1997) [23]	present work		
Newtonian fluid	48.12	47.3	46.19	47.91	75.21	90.05
Pseudoplastic fluid (n = 0.8)	51.04	----	----	51.00	80.13	105.26
Dilatant fluid (n = 1.2)	46.01	----	----	45.62	69.42	83.31

Fig.(7) shows the distribution of axial velocity with increasing the Hartmann number (M) for Newtonian fluid (Re=150) on lower and upper walls. When exposing the fluid flow in the channel to a certain magnetic field, there will be a electromagnetic force affecting the behavior of the fluid flow.

For low intensity magnetic field, the electromagnetic force is weak and this force has only a slight effect on the flow and inertia force remain is strong, Fig.7(b). But with increasing intensity magnetic field, the force of inertia will be diluted while the electromagnetic force becomes stronger, (Fig.7(c)). The velocity gradient increases in the side walls due to the combined effect of the Lorentz force and the no slip condition while reduces in the core. As we can see that the velocities in a channel are decrease with the increasing of the intensity magnetic fields.

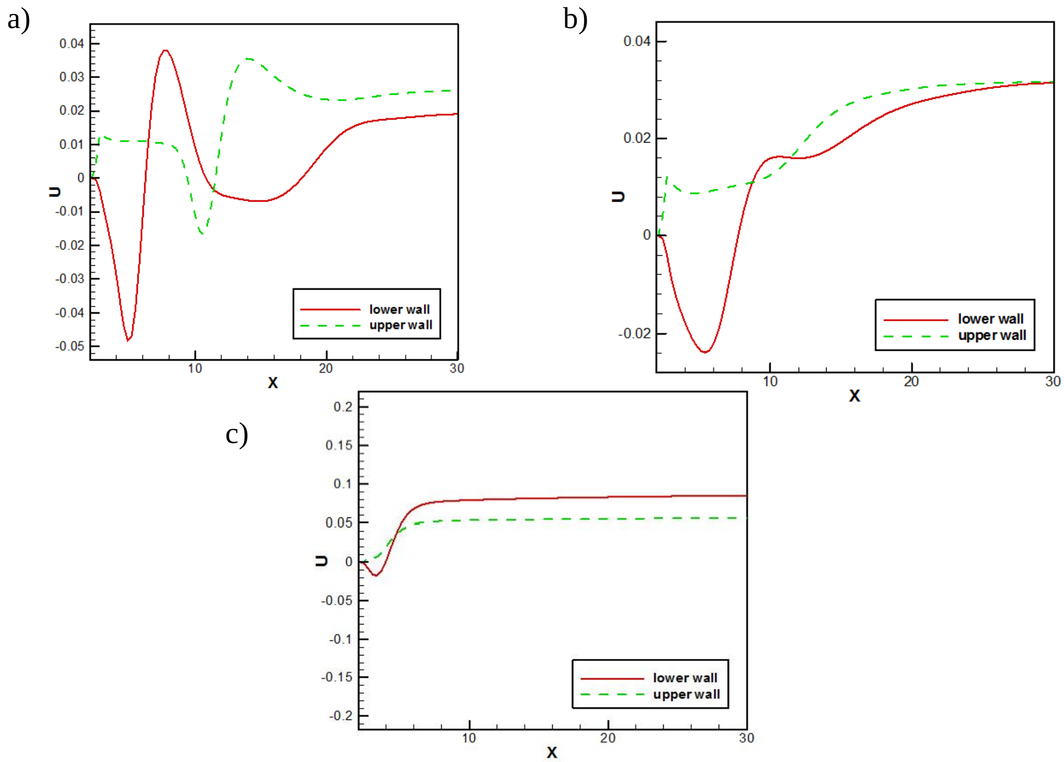


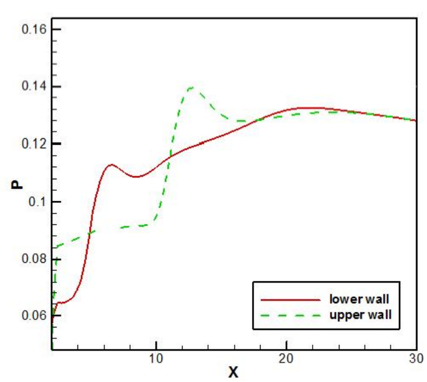
Figure (7): Distribution of axial velocity for Newtonian fluid on lower and upper walls with Re=150 and for various Hartmann numbers (M).
((a)M=0, (b) M=3, (c) M=7).

From the Fig.8 it is noticed that the effect of supplied the magnetic field on distribution of pressure for Newtonian fluid on upper and lower walls at Re=150.

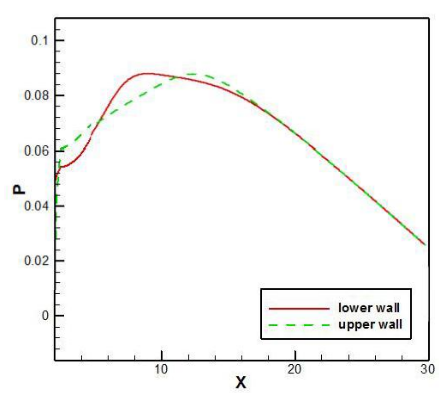
The pressure distribution has different values on the lower and upper walls before eventually reaching the same values and the constant slope for fully developed conditions, Fig.8a.

But with increasing Hartmann number (M=3) a pressure drop occurs downstream in the expansion part of the channel on the upper and lower walls, and with (M=7) the pressure increases slowly and reaches a maximum value in the beginning of the expansion part and then gradually decreases and down toward to end of the channel and here the flow become symmetric at M=7, Fig. 8(c).

a)



b)



c) `

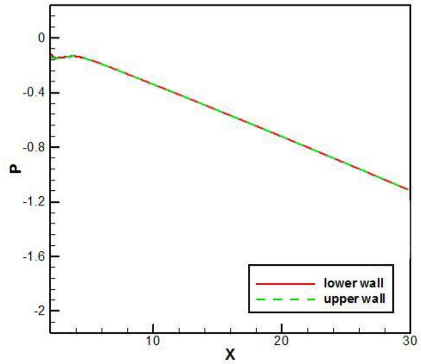


Figure (8): Distribution of pressure for Newtonian fluid on lower and upper walls with Re=150 and for various Hartmann numbers (M),
((a)M=0, (b) M=3, (c) M=7).

The evaluate the distribution of wall shear stress on the lower and upper walls is important in flow devices. Fig. 9 Distribution of wall shear stress for Newtonian fluid on upper and lower walls for various Hartmann numbers M=0, 3 and 7), at the Reynolds number Re=150. The recirculation lengths and the separation point on the lower and upper walls of the separation regions decrease with increasing supplied the external magnetic field. This occurs because of the influence of electromagnetic force acts to suppress the flow, as shown in Fig. 9a.

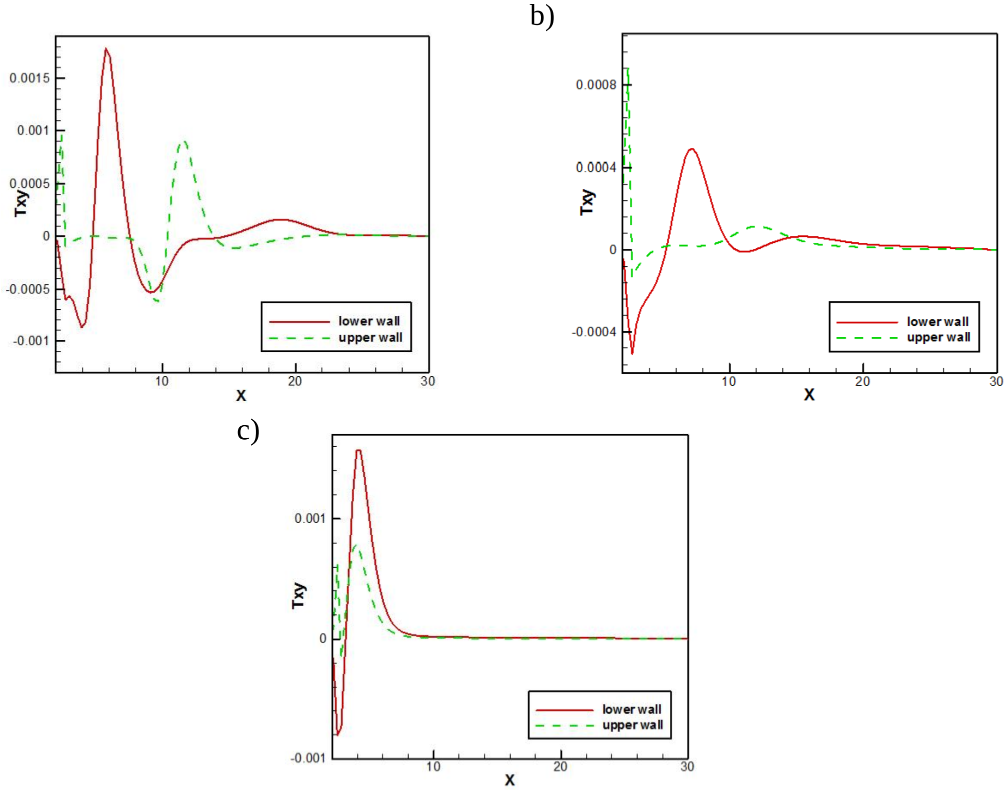


Figure (9): Distribution of wall shear stress for Newtonian fluid on lower and upper walls with Re=150 and for various Hartmann numbers (M).
((a)M=0, (b) M=3, (c) M=7).

Fig.s (10-12) show distribution of axial velocity on upper wall, lower wall and for the centerline of x-axis for Newtonian and Non-Newtonian fluid for various Hartmann numbers (M=0, 1 and 3) at Re=150. In the expanded channel, the distribution of axial velocity become oscillatory, and with increasing magnetic field (M), this oscillatory decreases and become constant. This shows that the expansion part of the channel is the source of the vortices and non-symmetric fluid flow.

In Fig.s (13-15) we see the effect wall shear stress distribution has been the same effect the axial velocity as shown in Fig.s(10-12). The distribution of wall shear stress become oscillatory in the expanded channel, and with increasing magnetic field (M), this oscillatory decreases and become constant

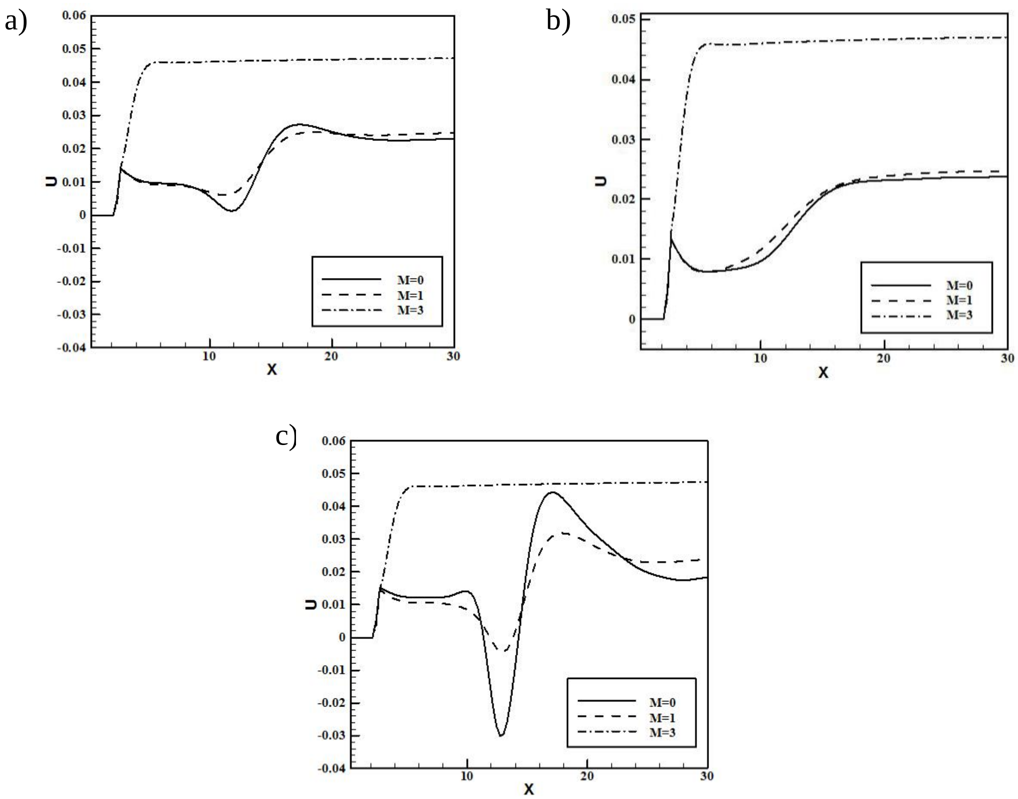
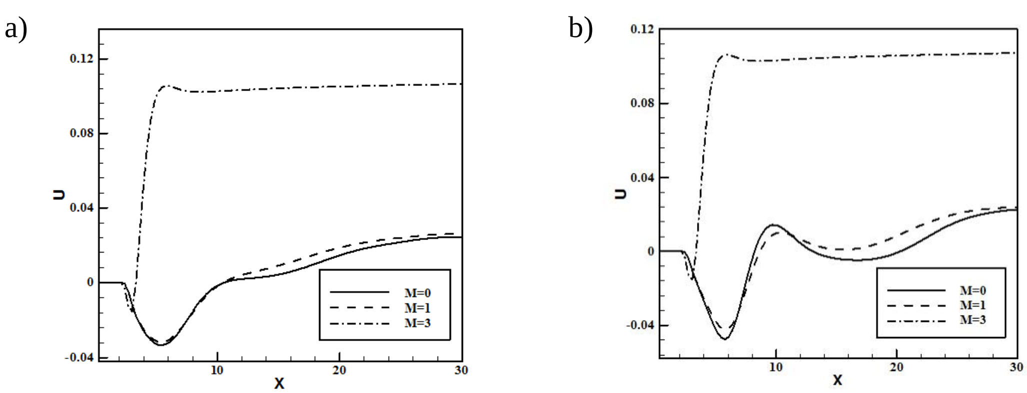


Figure (10): Distribution of axial velocity on upper wall for various Hartmann numbers and Re=150 for: a) n=0.8, b) n=1, c) n=1.2



c)

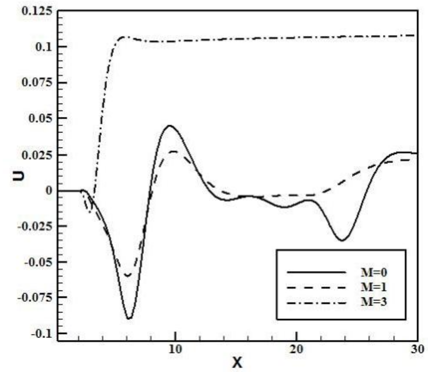
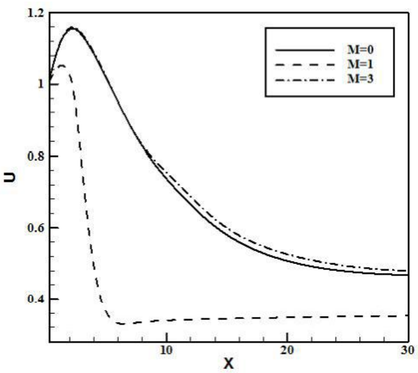
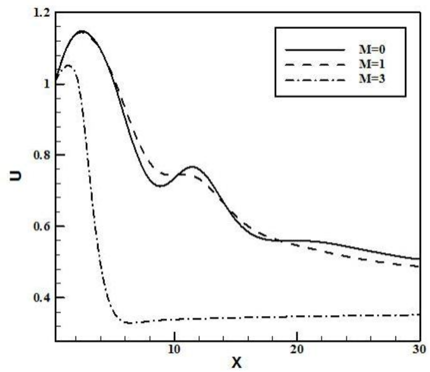


Figure (11): Distribution of axial velocity on lower wall for various Hartmann numbers and Re=150 for: a) n=0.8, b) n=1, c) n=1.2

a)



b)



c)

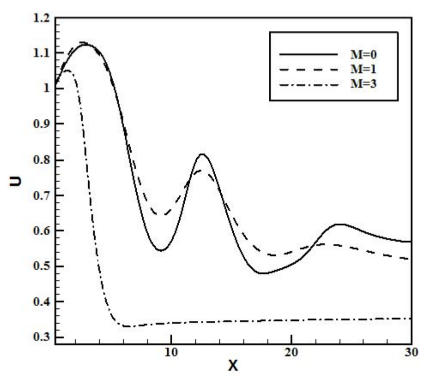
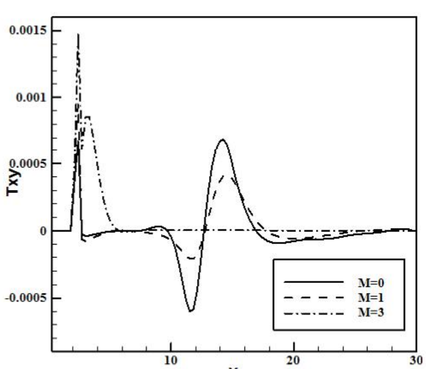
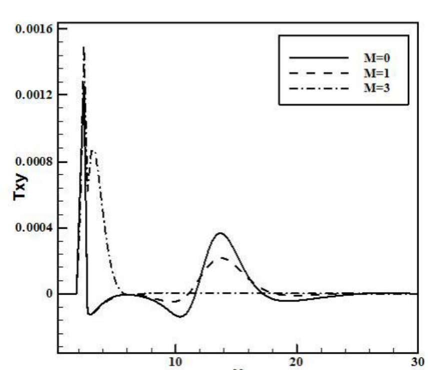


Figure (12): Distribution of axial velocity on center x-line for various Hartmann numbers and Re=150 for: a) n=0.8, b) n=1, c) n=1.2

a)



b)



c)

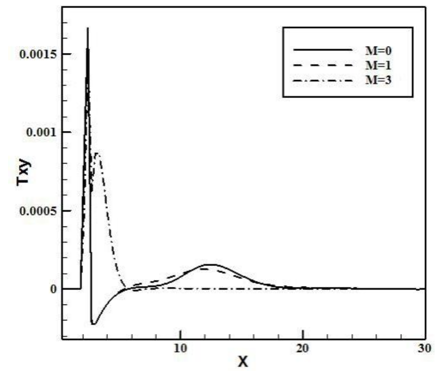
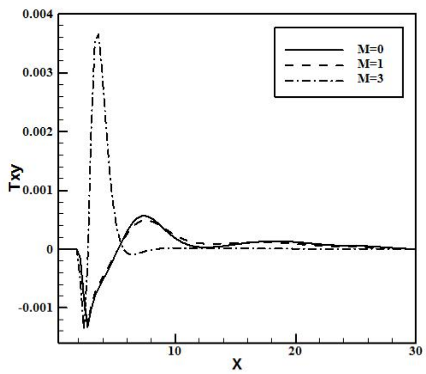
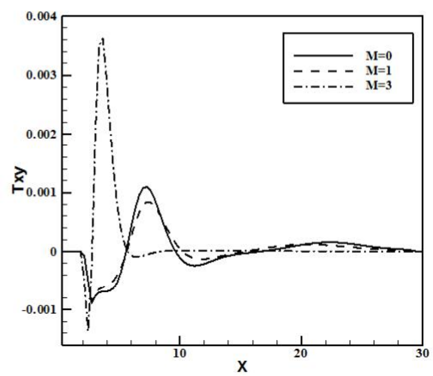


Figure (13): Distribution of wall shear stress on upper wall for various Hartmann numbers and Re=150 for: a) n=0.8, b) n=1, c) n=1.2

a)



b)



c)

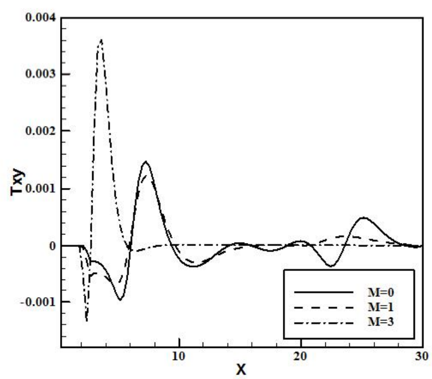
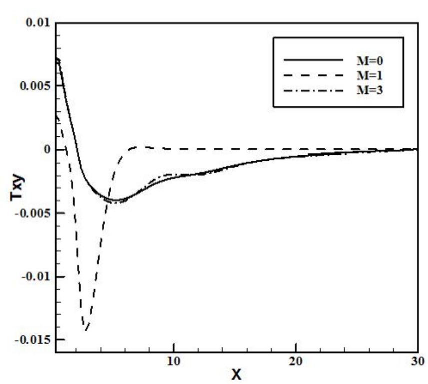
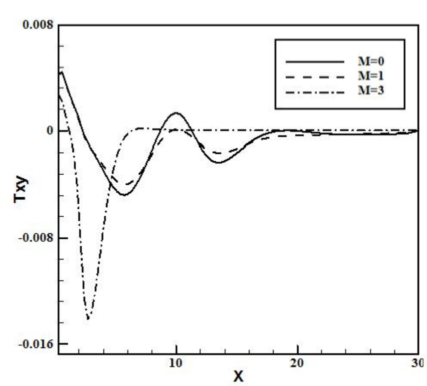


Figure (14): Distribution of wall shear stress on lower wall for various Hartmann numbers and Re=150 for: a) n=0.8, b) n=1, c) n=1.2

a)



b)



c)

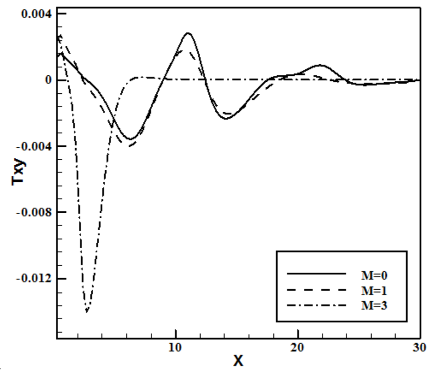


Figure (15): Distribution of wall shear stress on center x-line for various Hartmann numbers and Re=150 for: a) n=0.8, b) n=1, c) n=1.2

Conclusions

The effect of transverse uniform externally applied magnetic field on the flow of generalised Newtonian fluid (power-law model) in a two-dimensional 1:3 planar sudden expansion are investigated.

The magnetohydrodynamic equations are solved numerically using PISO-GNFMHD algorithm, which was developed by the author to include generalized Newtonian fluids, as well as the effects of the magnetic field on the fluid flow based on the finite volume method. The influence of various parameters such as the critical Reynolds number, power-law index and Hartmann number were studied in this numerical simulations.

The results of the present numerical model were compared in order to assess the accuracy of the numerical results with other experimental and numerical results and the comparison show a good agreement.

The following are some of the important conclusions of this paper:

1. The critical Reynolds number (Re_{cr}) is observed increases as power law index (n) decreases.
2. The critical Reynolds number (Re_{cr}) is delayed for shear-thinning fluids while the same occurs earlier for shear-thickening fluids compared to Newtonian fluids.
3. After occurrence of the bifurcation, the length of the lower primary vortex (Xr_2) is smaller than that of the upper vortex (Xr_1).
4. Fluid flow can be symmetrically at high values of Reynolds number by a suitable magnetic field and whenever the magnetic field (Hartmann number) applied to the fluid flow increases, the flow gradually returns to symmetric flow.

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