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Two parametric SEE transformation and its applications in solving differential equations

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ABSTRACT

Transformation plays a much more important role in every science. In this research article, two parametric forms of SEE transformation have been explored and the fundamental properties of two parametric SEE transformations have been shown. Furthermore, the transformed function of some fundamental functions and their time derivative rule has been shown. The application of two parametric SEE transformations in solving differential equations has been shown. The radioactive decay problem in first-order linear differential equations has been solved in this article which has large applications in nuclear energy engineering. Further, the solution to the beam deflection problem has been shown to have many applications in the engineering field. These results can be compared with other Laplace-type transformations.

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1. Introduction

The integral transformations map a complicated function from a functional space into a simple function in a transformed space. Where the function is characterized easily and manipulated through integration in transformed function space. To convert the transformed function into its original space often inverse transform techniques are used. The information on the topic is not lacking but the derivation of various research is not reliable and shows contrasts answered to the material utilized and area of analysis. A few studies from writing are assessed here which are related to the subject. The theory of Fourier and Laplace transformations was presented by many mathematicians, but no one has compared k –Fourier and k –Laplace transforms. The work on modern theories such as Integral transformation, Laplace transformation, Fourier transformation, Mahgoub transformation, Mohand transformation, and Aboodh transformation is still in progress. The authors have done a comparative

study of these transformations and proved that they are integral for solving many advanced problems in engineering and sciences.

In [1] a comparative study of the Adomian polynomial decomposition and approaches was made to finding Volterra differential-integral equations. The first and second types of non-linear Volterra differential-integral equations were solved analytically. In [2] by using an arrangement of the Laplace transform approach and the Adomian polynomial decomposition technique, it has been observed that Differential-integral equations rely on Lagrange interpolation. It was investigated by Rashed [3]. The Wavelet-Galerkin approach was used by the author [4]. The main purpose of this work was to find differential-integral equations results. To solve integral and differential-integer non-linear equations conducted a comparative analysis, Wazwaz [5] employed found many effective techniques. The Laplace transform method, which Khuri first examined in [6] and was

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presented and after some time updated. According to the new version it was combined with the Adomian polynomial decomposition technique to find the solution of differential non-linear equations. The technique for finding linked partial differential non-linear equations was examined by authors in [7]. For the solution of the Skan-Falkner equation, which describes two-dimensional incompressible laminar boundary layer equations, Elgasery [8] used the Laplace decomposition approach. In [9], the logistic differential equations were solved numerically with the help of the Laplace decomposition technique. The Adomian polynomial decomposition technique was studied by Chanqing and Jianhua in [10] for the meaning of the results of fractional differential non-linear equations.

The technique was used with delay differential equations in [11]. The differential-integral equations in [12] were resolved by using a modified Laplace series results via the Adomian polynomial decomposition method (LADM). The conventional Adomian polynomials were utilized to get around the non-linearity. To find out the numerical approximated results of the system of partial on linear differential equations in [13] practiced the Laplace decomposition approach and the Pade approximation. Additionally, It modified the Laplace decomposition method which was used in [14] for differential equalities of the Emden-Lane type. The first and second kinds of non-linear Volterra-Fredholm integration differential equations were solved analytically by the author [15] and using a collective form of the modified Laplace Adomian substitution and Laplace Adomian polynomial decomposition method (LADM). There are many applications for partial differential equations in the sciences. There are many different ways to solve partial differential equations. Engineering and scientific sectors frequently employ used Laplace integral techniques. Numerous writers [16-19] have discussed how integral transformations can be used to resolve improper integrals that included error functions. [20] Used the Kamal integral technique to find the answers to Abel's equation. [21] offered a Kamal integral technique-based solution to the error function.

In [22], the authors described the substitution method of Laplace transforms to solve linear partial differential equations with more than two independent variables. The influence of Bio-convection and activation energy on the Maxwell equation on nano-fluid has been explained by [23] with the help of the Matlab program. An analytical solution of the advection-diffusion equation in one-dimensional with a semi-infinite medium has been elaborated by [24] with the help of the Laplace technique of transformation. In 2021, the research in [25,26] introduced a new transformation to solve a new type of differential equations with trigonometric coefficients. Application of Kamal transformation in thermal engineering has been shown in [27], the solution of temperature problem and some of its applications shown in the article. In [28] a new transformation named SEE transformation has been shown which is quite helpful in solving differential equations and systems of differential equations. The solution of differential equations of moment Pareto distribution with logarithmic coefficients has been shown in [29] with the help of the Al-Zughair transformation. A new transformation in the logarithmic kernel has been shown in the article [30], this transformation helps solve differential equations with logarithmic coefficients and also ordinary differential equations.

In this article, we are going to introduce two parametric SEE transformations which are the generalized form of SEE transformation, and their applications in engineering are shown in this research.

2. Theoretical analysis

Two parametric SEE transformations can be defined below.

Def. 1: Two parametric SEE integral transform of the function $f(t)$ for all $t \geq 0$ is defined as:

$$SEE_{\alpha,\beta}\{f(t)\} = \frac{e^{-\beta v}}{(\alpha v)^n} \int_0^\infty f(t)e^{-(\alpha v)t} dt = F_{\alpha,\beta}(v) \tag{1}$$

Where α, β are real numbers, n is the integer and v is the transformed variable. By choosing the values of parameters α, β as arbitrary real numbers and integer n , we can get the different transformations as Laplace, Kamla, Mohand, SEE, Sumudu, Aboodh, and many other Laplace-type transformations.

Def. 2: The inverse of $F_{\alpha,\beta}(v)$ can be written as,

$$SEE_{\alpha,\beta}^{-1}\{F_{\alpha,\beta}(v)\} = f(t) \tag{2}$$

Table 1. Two parametric SEE transformations of some function

Function $f(t)$	$SEE_{\alpha,\beta}\{f(t)\}$ $= \frac{e^{-\beta v}}{(\alpha v)^n} \int_0^\infty f(t)e^{-(\alpha v)t} dt$
k	$\frac{e^{-\beta v} k}{(\alpha v)^{n+1}}$
1	$\frac{e^{-\beta v}}{(\alpha v)^{n+1}}$
t	$\frac{e^{-\beta v}}{(\alpha v)^{n+2}}$
t^2	$\frac{e^{-\beta v} 2!}{(\alpha v)^{n+3}}$
t^m	$\frac{e^{-\beta v} m!}{(\alpha v)^{n+m+1}}$
e^{ut}	$\frac{e^{-\beta v}}{(\alpha v)^n (\alpha v - u)}$
$\sin(ut)$	$\frac{e^{-\beta v}}{(\alpha v)^n} \left[\frac{u}{(\alpha v)^2 + u^2} \right]$
$\cos(ut)$	$\frac{e^{-\beta v}}{(\alpha v)^n} \left[\frac{\alpha v}{(\alpha v)^2 + u^2} \right]$
$\cosh(ut)$	$\frac{e^{-\beta v}}{(\alpha v)^n} \left[\frac{u}{(\alpha v)^2 + u^2} \right]$
$\sinh(ut)$	$\frac{e^{-\beta v}}{(\alpha v)^n} \left[\frac{\alpha v}{(\alpha v)^2 - u^2} \right]$

Theorem: Two parametric SEE transform for first, second up-to nth order derivative can be described as:

$$\begin{aligned} \text{I)} \quad & SEE_{\alpha,\beta}\{f'(t)\} = -\frac{e^{-\beta v}}{(\alpha v)^n} f(0) + (\alpha v)F_{\alpha,\beta}(v) \\ \text{II)} \quad & SEE_{\alpha,\beta}\{f''(t)\} = -\frac{e^{-\beta v}}{(\alpha v)^n} f'(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}} f(0) \\ & + (\alpha v)^2 F_{\alpha,\beta}(v) \end{aligned}$$

$$\begin{aligned}
 \text{III)} \quad SEE_{\alpha,\beta}\{f'''(t)\} &= -\frac{e^{-\beta v}}{(\alpha v)^n} f'''(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}} f'(0) \\
 &\quad - \frac{e^{-\beta v}}{(\alpha v)^{n-2}} f(0) + (\alpha v)^3 F_{\alpha,\beta}(v) \\
 \text{IV)} \quad SEE_{\alpha,\beta}\{f^{(m)}(t)\} &= -\frac{e^{-\beta v}}{(\alpha v)^n} f^{(m-1)}(0) \\
 &\quad - \frac{e^{-\beta v}}{(\alpha v)^{n-1}} f^{(m-2)}(0) \\
 &\quad - \frac{e^{-\beta v}}{(\alpha v)^{n-2}} f^{(m-3)}(0) - \dots \\
 &\quad - \frac{e^{-\beta v}}{(\alpha v)^{n-m+1}} f(0) \\
 &\quad + (\alpha v)^m SEE_{\alpha,\beta}\{f(t)\}
 \end{aligned}$$

Proof (1): Let double parametric SEE transformation for $f(t) = f'(t)$

$$\begin{aligned}
 SEE_{\alpha,\beta}\{f'(t)\} &= \frac{e^{-\beta v}}{(\alpha v)^n} \int_0^\infty f'(t) e^{-(\alpha v)t} dt \\
 &= \frac{e^{-\beta v}}{(\alpha v)^n} f(t)|_0^\infty + (\alpha v) \int_0^\infty f(t) e^{-(\alpha v)t} dt \\
 &= -\frac{e^{-\beta v}}{(\alpha v)^n} f(0) + (\alpha v) F_{\alpha,\beta}(v)
 \end{aligned} \quad (3)$$

Proof (2): Let double parametric SEE transformation for $f(t) = f''(t)$

$$\begin{aligned}
 SEE_{\alpha,\beta}\{f''(t)\} &= \frac{e^{-\beta v}}{(\alpha v)^n} \int_0^\infty f''(t) e^{-(\alpha v)t} dt \\
 &= \frac{e^{-\beta v}}{(\alpha v)^n} f'(t)|_0^\infty + (\alpha v) \int_0^\infty f'(t) e^{-(\alpha v)t} dt \\
 &= -\frac{e^{-\beta v}}{(\alpha v)^n} f'(0) + (\alpha v) SEE_{\alpha,\beta}\{f'(t)\}
 \end{aligned}$$

With the help of eq. 3 we can get,

$$= -\frac{e^{-\beta v}}{(\alpha v)^n} f'(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}} f(0) + (\alpha v)^2 F_{\alpha,\beta}(v) \quad (4)$$

Proof (3): Let double parametric SEE transformation for $f(t) = f'''(t)$

$$\begin{aligned}
 SEE_{\alpha,\beta}\{f'''(t)\} &= \frac{e^{-\beta v}}{(\alpha v)^n} \int_0^\infty f'''(t) e^{-(\alpha v)t} dt \\
 &= \frac{e^{-\beta v}}{(\alpha v)^n} f''(t)|_0^\infty + (\alpha v) \int_0^\infty f''(t) e^{-(\alpha v)t} dt \\
 &= -\frac{e^{-\beta v}}{(\alpha v)^n} f''(0) + (\alpha v) SEE_{\alpha,\beta}\{f''(t)\}
 \end{aligned}$$

With the help of eq. 4 we can get,

$$= -\frac{e^{-\beta v}}{(\alpha v)^n} f''(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}} f'(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-2}} f(0) + (\alpha v)^3 F_{\alpha,\beta}(v) \quad (5)$$

Similarly, we can find the result from statement 4 mentioned in the theorem,

$$\begin{aligned}
 SEE_{\alpha,\beta}\{f^{(m)}(t)\} &= -\frac{e^{-\beta v}}{(\alpha v)^n} f^{(m-1)}(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}} f^{(m-2)}(0) - \\
 &\quad - \frac{e^{-\beta v}}{(\alpha v)^{n-2}} f^{(m-3)}(0) - \dots - \frac{e^{-\beta v}}{(\alpha v)^{n-m+1}} f(0) + (\alpha v)^m SEE_{\alpha,\beta}\{f(t)\}
 \end{aligned} \quad (6)$$

Linearity property:

Let $f(t)$ and $g(t)$ be two functions than the linearity property of two parametric extensions of SEE transformation define as

$$SEE_{\alpha,\beta}\{c_1 f(t) \pm c_2 g(t)\} = c_1 SEE_{\alpha,\beta} f(t) \pm c_2 SEE_{\alpha,\beta} g(t) \quad (7)$$

Where c_1 and c_2 are constants.

Change of scale property:

Let $f(t)$ be a function and $SEE_{\alpha,\beta}$ of $f(t)$ is $F_{\alpha,\beta}(v)$ then the change of scale property of two parametric SEE transformations can be described as,

$$SEE_{\alpha,\beta}\{f(ct)\} = \left(\frac{1}{c^{n-1}}\right) F_{\alpha,\beta}\left(\frac{v}{c}\right) \quad (8)$$

Here c is a constant value.

Shifting Property:

Let $f(t)$ be a function and $SEE_{\alpha,\beta}$ of $f(t)$ is $F_{\alpha,\beta}(v)$ then shifting property of two parametric SEE transformations can be described as,

$$SEE_{\alpha,\beta}\{e^{ct} f(t)\} = F_{\alpha,\beta}(v - c) \quad (9)$$

Convolution Theorem:

Let $f(t)$ and $g(t)$ be two functions and $SEE_{\alpha,\beta}$ of $f(t)$ is $F_{\alpha,\beta}(v)$, $SEE_{\alpha,\beta}$ of $h(t)$ is $H_{\alpha,\beta}(v)$ then the convolution of two parametric SEE transforms can be described as,

$$SEE_{\alpha,\beta}\{f(t) * h(t)\} = -(\alpha v)^n F_{\alpha,\beta}(v) \cdot H_{\alpha,\beta}(v) \quad (10)$$

3. Applications

Two parametric SEE is used to Solve the radioactive decay problem: Exponential decay is linked to the process of radioactive decay and population decline. The half-life of a substance is the amount of time it takes for half of it to deteriorate or disintegrate. [31] Has mathematically described the decay problems of a substance with the help of first-order ordinary linear differential equation,

$$\frac{dN}{dt} = -\mu N \quad (11)$$

With the initial condition,

$$N(0) = N_0 \quad (12)$$

Where μ is a real positive number, N is the value of population at t -time and N_0 is the initial value of the population at the initial time t_0 .

Solution: By applying the two parametric SEE transform on eq. 11 and by using the rule mentioned in eq. 3 we will get,

$$-\frac{e^{-\beta v}}{(\alpha v)^n} N(0) + (\alpha v) N_{\alpha,\beta}(v) = -\mu N_{\alpha,\beta}(v)$$

By using the initial condition in eq. 12 we get,

$$\begin{aligned}
 -\frac{e^{-\beta v}}{(\alpha v)^n} N_0 + (\alpha v) N_{\alpha,\beta}(v) &= -\mu N_{\alpha,\beta}(v) \\
 [(\alpha v) + \mu] N_{\alpha,\beta}(v) &= \frac{e^{-\beta v}}{(\alpha v)^n} N_0 \\
 N_{\alpha,\beta}(v) &= \frac{e^{-\beta v}}{(\alpha v)^n [(\alpha v) + \mu]} N_0
 \end{aligned} \quad (13)$$

By using the definition in eq. 2 and table values we get,

$$N(t) = N_0 e^{-\mu t} \quad (14)$$

Equation 14 provides the solution to the decay problem.

Two parametric SEE to Solve the Beam Deflecting problem:

Let a beam with uniform load $q(x)$ be hung by two supports at the end. The deflection of $y(x)$ is explained by the given differential equation along x – $axis$.

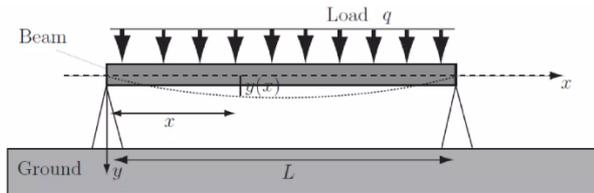


Figure1. Beam deflection with hanging uniform load

$$\frac{d^4y}{dx^4} = \frac{1}{EI}q(x) \quad 0 < x < L \tag{15}$$

Where

E is the elastic modulus

I is the moment of inertia

EI is the flexural rigidity of the beam

With the given condition

$$\left. \begin{matrix} y(0) = 0 \\ y(L) = 0 \end{matrix} \right\} \text{no deflection.} \tag{16}$$

$$\left. \begin{matrix} y''(0) = 0 \\ y''(L) = 0 \end{matrix} \right\} \text{no bending.} \tag{17}$$

The $q(x) = q$ as a constant load

Solution: Apply the two parametric SEE transformations on Eq. 15,

$$SEE_{\alpha,\beta} \left\{ \frac{d^4y}{dx^4} \right\} - SEE_{\alpha,\beta} \left\{ \frac{1}{EI}q \right\} = 0$$

By using the def. of two parametric SEE transformations and rule in Eq.6, to get,

$$-\frac{e^{-\beta v}}{(\alpha v)^n} y'''(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-1}} y''(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-2}} y'(0) - \frac{e^{-\beta v}}{(\alpha v)^{n-3}} y(0) + (\alpha v)^4 Y_{\alpha,\beta}(v) - \frac{q}{EI} \frac{e^{-\beta v}}{(\alpha v)^{n+1}} = 0 \tag{18}$$

Applying the condition provided in Eq. 16 and 17, where let the unknown $y'(0) = \omega_1$ and $y'''(0) = \omega_2$

$$\begin{aligned} -\frac{e^{-\beta v}}{(\alpha v)^n} \omega_2 - \frac{e^{-\beta v}}{(\alpha v)^{n-2}} \omega_1 + (\alpha v)^4 Y_{\alpha,\beta}(v) &= \frac{q}{EI} \frac{e^{-\beta v}}{(\alpha v)^{n+1}} \\ (\alpha v)^4 Y_{\alpha,\beta}(v) &= \frac{q}{EI} \frac{e^{-\beta v}}{(\alpha v)^{n+1}} + \frac{e^{-\beta v}}{(\alpha v)^n} \omega_2 + \frac{e^{-\beta v}}{(\alpha v)^{n-2}} \omega_1 \\ Y_{\alpha,\beta}(v) &= \frac{q}{EI} \frac{e^{-\beta v}}{(\alpha v)^4(\alpha v)^{n+1}} + \frac{e^{-\beta v}}{(\alpha v)^4(\alpha v)^n} \omega_2 + \frac{e^{-\beta v}}{(\alpha v)^4(\alpha v)^{n-2}} \omega_1 \end{aligned} \tag{19}$$

By using the inverse of two parametric SEE transformations on eq. 19 we get,

$$y(x) = \frac{qx^4}{24EI} + \frac{\omega_2}{6}x^3 + \omega_1x \tag{20}$$

The unknowns can be found by using the boundary conditions given in Eq. 16 and 17, we can get,

$$\begin{aligned} \omega_2 &= -\frac{q}{2EI}L; & \omega_1 &= \frac{q}{24EI}L^3 \\ y(x) &= \frac{qx^4}{24EI} - \frac{q}{12EI}Lx^3 + \frac{q}{24EI}L^3x \end{aligned} \tag{21}$$

Eq. 21 provides the solution to the deflection of the beam problem hung by ending points.

4. Conclusion

In this research, the suggested technique is the generalized form of SEE transformation. Presented two parametric SEE transformation is quite important and efficient in solving the differential equation. This general form can be turned into ordinary form by fixing the parameters. The solution of decay problems and beam deflection problems with uniform load is presented in this study which has a large application in engineering fields. This technique is comparable with other Laplace-type transformations.

Authors' contribution

All authors contributed equally to the preparation of this article.

Declaration of competing interest

The authors declare no conflicts of interest.

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