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An optimization model for the layout of a branching water distribution system

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ABSTRACT

This paper presents an optimization model for the layout of a branching water distribution system. The goal is to minimize construction costs while meeting the system's demands. The study utilizes the general algebraic modeling system (GAMS) to optimize a non-looping water distribution system. The methodology involves determining the existence and diameter of connections between demand nodes. The optimization problem is formulated as a mixed-integer non-linear programming (MINLP) problem. A simplified layout is used to illustrate the constraints and validate the model. The explicit model implemented in GAMS yields optimal solutions and demonstrates the effectiveness of the approach. The results highlight the decisions on connection existence, flow, and pipe diameter, contributing to cost minimization. The findings from this study provide insights for optimizing the design of branching water distribution systems and reducing construction costs.

1. Introduction

Branching water distribution systems can be found in small systems like irrigation to large-scale deliverance of water supply to different cities. The cost of constructing these systems is related to both the size of the connecting pieces and the length of those connections between delivery points. In order to minimize construction costs, these parameters must be optimized in a way that still meets the required demands of the system. The objective of this project is to use the general algebraic modeling system (GAMS) to optimize the layout of a non-looping water distribution system such that cost is minimized. This project will look at a dendritic water supply pipe system consisting of multiple nodes, *m*, on a variety of Isonodal lines, *n*, as shown in Fig. 1.

The dotted lines running between the Isonodal lines represent possible connections between the upstream nodes *mn* and downstream nodes *mn+1*. The objective of this project is to determine whether those connections should exist and, if so, what the diameter should be so that demands are met while capital costs are minimized.

2. Methodology

The unique aspect of this project is determining whether a connection, in this case pipe, exists between two demand nodes. A variable $\boldsymbol{a_{n,m_n,m_{n+1}}}$ will be set at “0” or “1” to show if the connection (from *mn* online *n* to

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m_{n+1} on line $n+1$) is not allowed or allowed, respectively. A zero value may indicate that the connections are too far apart or that some physical impediment exists to construction in the field. A related decision variable $\boldsymbol{x_{n,m_n,m_{n+1}}}$ will be sought to determine whether the connection is optimal for existence. These will, respectively, be represented by a "1" or "0". As the nodes are all at fixed locations, the lengths $\boldsymbol{L_{n,m_n,m_{n+1}}}$ of candidate pipes can be determined. Each node will have a required flow $\boldsymbol{Q_{n,m_n}}$ and minimum pressure head $\boldsymbol{H_{n,m_n}}$. The flows through the pipes $\boldsymbol{QP_{n,m_n,m_{n+1}}}$ will be the sum of all the demands downstream of the starting node of the pipe. A Darcy-Weisbach friction factor will be used to find losses due to pressurized flow. These variables will be used to determine another decision variable, the diameter $\boldsymbol{D_{n,m_n,m_{n+1}}}$ of that pipe, which in turn determines the pipe cost per unit length $\boldsymbol{C(D_{n,m_n,m_{n+1}})}$. **Heat losses** through the pipe are non-linear**,** while both $\boldsymbol{a_{n,m_n,m_{n+1}}}$ and $\boldsymbol{x_{n,m_n,m_{n+1}}}$ are integer values.

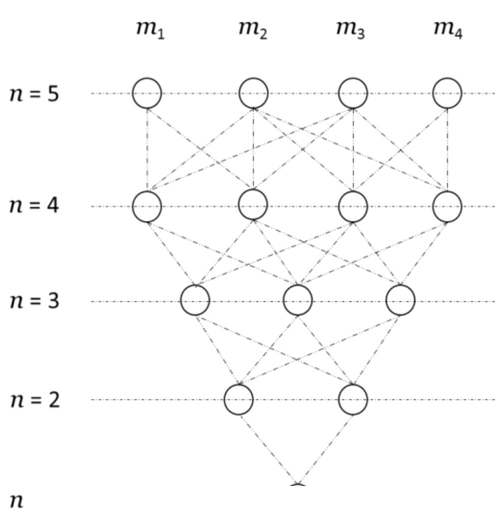

Figure 1. General system layout

This makes the optimization a mixed-integer non-linear programming (MINLP) problem.

Defined Variables:

$a_{n,m_n,m_{n+1}}$: Allowance of connection between nodes m_n and m_{n+1} (0 or 1)
$L_{n,m_n,m_{n+1}}$: The length of pipe between nodes m_n and m_{n+1}
$J_{n,m_n,m_{n+1}}$: Headless per unit length of pipe connecting nodes m_n and m_{n+1}
H_{n,m_n} : Required head at node m_n
Q_{n,m_n} : Required flow at node m_n
P_s : Starting pressure head (elevation) of the system

Decision Variables:

$x_{n,m_n,m_{n+1}}$: Existence of connection between nodes m_n and m_{n+1} (binary variable: 0 or 1)
$QP_{n,m_n,m_{n+1}}$: Flow through pipe between nodes nodes m_n and m_{n+1}
$D_{n,m_n,m_{n+1}}$: Pipe diameter for a given link between nodes m_n and m_{n+1}

The objective function is given as:
Minimize

$$Z = \sum_n \sum_{m_n} \sum_{m_{n+1}} x_{n,m_n,m_{n+1}} L_{n,m_n,m_{n+1}} C(D_{n,m_n,m_{n+1}}) \quad (1)$$

Where:

$$C = 11.7 + 0.51 D_{n,m_n,m_{n+1}}{}^{1.38}$$

Given for 12-54 in concrete piping material costs from Clark et al. Subject to:

Connectivity:

$$\sum_{m_n} a_{n,m_n,m_{n+1}} x_{n,m_n,m_{n+1}} = 1 \qquad \forall\, n+1 \text{ and } \forall\, m_{n+1} \quad (2)$$

Conservation of flow:

$$QP_{n,m_n,m_{n+1}} = \left[\sum_{m_{n+1}} QP_{n+1,m_{n+1},m_{n+2}} X_{n+1,m_{n+1},m_{n+2}} + Q_{n+1,m_{n+1}}\right] x_{n,m_n,m_{n+1}} \quad (3)$$

Conservation of energy:

$$H_{n+1,m_{n+1}} \le H_{1,1} - \sum_n \sum_{m_n} \sum_{m_{n+1}} L_{n,m_n,m_{n+1}} J_{n,m_n,m_{n+1}} X_{n,m_n,m_{n+1}} \quad (4)$$

Where,

$$J_{n,m_n,m_{n+1}} = \frac{8 f\, QP_{n,m_n,m_{n+1}}{}^2}{\pi^2 g D_{n,m_n,m_{n+1}}{}^5}$$

Non-negativity:

$$QP_{n,m_n,m_{n+1}},\, D_{n,m_n,m_{n+1}} \ge 0$$

2.1 Simple layout

A simplified version of the layout is used to illustrate the constraints which will be automatically delineated by GAMS in the final model. The layout is the first three Isonodal lines with possible connections varying by order of magnitude as shown in Fig. 2.

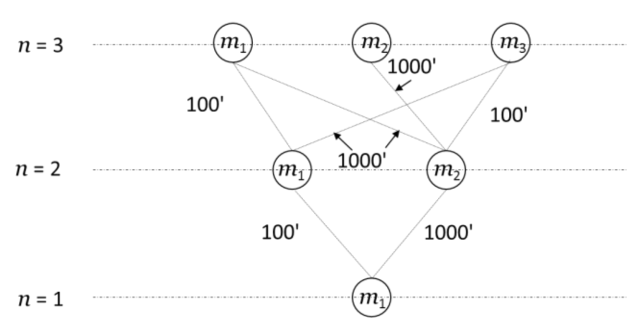

Figure 2. Simplified layout

Subject to:
Connectivity:

Node m_1 on n = 2 $\quad a_{1,1,1}x_{1,1,1} = 1$
Node m_2 on n = 2 $\quad a_{1,1,2}x_{1,1,2} = 1$
Node m_1 on n = 3 $\quad a_{2,1,1}x_{2,1,1} + a_{2,2,1}x_{2,2,1} = 1$
Node m_2 on n = 3 $\quad a_{2,1,2}x_{2,1,2} + a_{2,2,2}x_{2,2,2} = 1$
Node m_3 on n = 3 $\quad a_{2,1,3}x_{2,1,3} + a_{2,2,3}x_{2,2,3} = 1$

Conservation of flow:

$QP_{1,1,1} = [QP_{2,1,1}X_{2,1,1} + QP_{2,1,2}X_{2,1,2} + QP_{2,1,3}X_{2,1,3} + Q_{2,1}]x_{1,1,1}$
$QP_{1,1,2} = [QP_{2,2,1}X_{2,2,1} + QP_{2,2,2}X_{2,2,2} + QP_{2,2,3}X_{2,2,3} + Q_{2,2}]x_{1,1,2}$
$QP_{2,1,1} = [0 + Q_{3,1}]x_{2,1,1}$
$QP_{2,1,2} = [0 + Q_{3,2}]x_{2,1,2}$
$QP_{2,1,3} = [0 + Q_{3,3}]x_{2,1,3}$
$QP_{2,2,1} = [0 + Q_{3,1}]x_{2,2,1}$
$QP_{2,2,2} = [0 + Q_{3,2}]x_{2,2,2}$
$QP_{2,2,3} = [0 + Q_{3,3}]x_{2,2,3}$

Hydraulic Constraints:
$H_{2,1} \leq H_{1,1} - (L_{1,1,1}J_{1,1,1}X_{1,1,1})$
$H_{2,2} \leq H_{1,1} - (L_{1,1,2}J_{1,1,2}X_{1,1,2})$
$H_{3,1} \leq H_{1,1} - (L_{1,1,1}J_{1,1,1}X_{1,1,1} + L_{2,1,1}J_{2,1,1}X_{2,1,1})$
or $H_{3,1} \leq H_{1,1} - (L_{1,1,2}J_{1,1,2}X_{1,1,2} + L_{2,2,1}J_{2,2,1}X_{2,2,1})$
$H_{3,2} \leq H_{1,1} - (L_{1,1,1}J_{1,1,1}X_{1,1,1} + L_{2,1,2}J_{2,1,2}X_{2,1,2})$
or $H_{3,2} \leq H_{1,1} - (L_{1,1,2}J_{1,1,2}X_{1,1,2} + L_{2,2,2}J_{2,2,2}X_{2,2,2})$
$H_{3,3} \leq H_{1,1} - (L_{1,1,1}J_{1,1,1}X_{1,1,1} + L_{2,1,3}J_{2,1,3}X_{2,1,3})$
or $H_{3,3} \leq H_{1,1} - (L_{1,1,2}J_{1,1,2}X_{1,1,2} + L_{2,2,3}J_{2,2,3}X_{2,2,3})$

The source node, m_1 on $n = 1$, was given a total head of 550 ft. All other nodes were given a required head of 500 ft and a demand of 5 cfs. The friction factor f was set at 0.02. These values will be the same set-up in the larger scenario as well.

2.2 Explicit model of simple layout

The above equations were entered explicitly, or directly as-is, into GAMS to both ensure that the engineering is correct and to ascertain correct values to check the more generalized model against. The resulting values for the decisions variables are shown in Table 1, and an objective value of $65,897 was determined.

It should be noted that GAMS specifies diameters for pipes that do not exist. Since d is in the denominator of the headloss equation, a "division by zero" error occurs if they are set to zero. However, this does not affect the value of the objective function since they are multiplied by $x = 0$. The diameters are also not commercial sizes or even rounded up to the next inch. Setting D as an integer variable in GAMS returned an error of "no integer solution found" and gave the resulting values. The flows and existence were easily predicted based on the simple setup of the problem. However, the diameters were more difficult to verify, so the hydraulics were checked using EPANET. The heads at each node are shown in Figure 3.

Table 1. Explicit GAMS output for simple layout

Pipe	x	qp	d
1,1,1	1	10	08.473
1,1,2	1	15	15.619
2,1,1	1	05	06.818
2,1,2	0	00	01.000
2,1,3	0	00	01.830
2,2,1	0	00	00.814
2,2,2	1	05	10.980
2,2,3	1	05	06.928

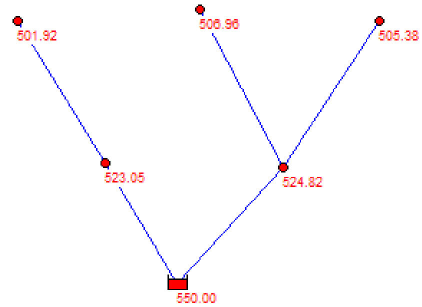

Figure 3. EPANET pressure heads of explicit simple system

All heads are above the required 500ft in addition to the flows and connections being what was expected. This shows that the equations are properly reflecting the engineering.

2.3 Generalized Model of Simple Layout

With validated equations and verified values, the next step was to generalize the GAMS model so that GAMS creates the constraints for each node and pipe from the general constraints instead of having to explicitly write them all out. The original idea was to use *n*, *mn*, and *mn*+1 for the sets as below:

n 5 Isonodal lines /n1, n2, n3/
mn 4 nodes on n /mn1, mn2, mn3/
mn1 4 nodes on n+1 /mn11, mn12, mn13/

This created problems with switching between Isonodal lines. Sets that needed to be summed over also needed to be controlled, causing errors in GAMS. Because of this, a new constraint equation or condition would be required for each Isonodal line or node position on the line. The flow equation, for example, would require a conditional constraint for each pipe flow and each node demand for each node position, or six conditions in all. This limited the amount of generalization that could be done and also meant more work for larger layouts. To circumvent this problem, the approach taken by Kurt Mahoney and Joshua Steele for their storm sewer layout model was adapted. This reduces the sets to one for Isonodal lines and one for nodes. Aliases are then used to help the equations sum over nodes on different Isonodal lines below:

n 3 Isonodal lines /n1, n2, n3/
mn 3 possible node on n /1, 2, 3/
alias (mn, mn1, mnm1, mnm2)

The connectivity constraint is relatively straightforward, though Mahoney and Steele added a condition to state that there are no connections if all $a_{n,m_n,m_{n+1}}$ equal 0, and the code does not work properly without it.

*Connectivity (n, mn1). sum (mn, x(n,mn,mn1)*a(n,mn,mn1)) =e=1$(sum (mn, a (n, mn, mn1))>0)*

Their approach also looked at *QP* in relation to the node where flow is exiting, or in this case entering, rather than the specific pipe itself. This allows GAMS to sum over the downstream pipes since *QP* is defined by (n, mn) instead of (n,mn,mn1) . Equation 3 then becomes:

$$QP_{n,m_n} = \left[\sum_{m_{n+1}} QP_{n+1,m_{n+1}} X_{n+1,m_{n+1},m_{n+2}} + Q_{n+1,m_{n+1}}\right] x_{n,m_n,m_{n+1}}$$

or in GAMS flow:

*(n,mn)..qp(n,mn)=e=q(n,mn)+sum(mn1,qp(n+1,mn1)*x(n,mn,mn1))$(ord (n)>1);*

This can cause a bit of confusion as QP_{n,m_n} could apply to multiple cases of $QP_{n,m_n,m_{n+1}}$.

However, this is easily handled by multiplying by $x_{n,m_n,m_{n+1}}$. Extra equations are included in the simple layout GAMS model for illustration purposes. In addition to the sets and flow equation, the model from Mahoney and Steele had to be expanded to account for pressurized flow and cost. As with *QP*, *D* was converted to a nodal designation instead of a specific pipe so that:

$$J_{n,m_n,m_{n+1}} = \frac{8fQP_{n,m_n}{}^2}{\pi^2 g D_{n,m_n}{}^5}$$

The hydraulic constraint is the most challenging because it crosses more than one pair of Isonodal lines as it traces headless from the source. This means that pathways must be considered backward from (n, mn) through (n-1,mn-1, mn) and (n-2,mn-2,mn-1). For example, Eq 4 becomes:

$$H_{3,1} \le H_{1,1} - \sum_{m_{n-1}} L_{n-1,m_n-1,m_1} J_{n-1,m_n-1,m_1} X_{n-1,m_n-1,m_1} - \sum_{m_{n-2}} L_{n-2,m_{n-2},m_{n-1}} J_{n-2,m_{n-2},m_{n-1}} X_{n-2,m_{n-2},m_{n-1}}$$

Under this method, each summation must be entered separately into GAMS.

hydraulic(n,mn)..h(n,mn)=l=h('n1','1')-coeffj (sum(mnm1,(l(n-1,mnm1,mn)*x(n 1,mnm1,mn)*qp(n,mn)**2/((d(n,mn)/12)**5))$(ord(n)>1) +sum(mnm2, (l(n-2,mnm2,mnm1)*x(n-2,mnm2,mnm1)*qp(n-1,mnm1)**2/((d(n-1,mnm1)/12)**5))$(ord(n)>2))));*

It is possible to look at headless between only two adjacent Isonodal lines. In this case, the head of the upstream node would be used instead of the head of the source and there would only be one connecting pipe. However, this requires an additional decision variable and constraint to account for the actual head at each node. (The main layout was already reaching GAMS demo limitations, though, so this would have been impractical in that regard). Finally, the objective function was added using the modified designation for *D*.

*mincost. sum(n, sum(mn, sum(mn1,x(n,mn,mn1)*l(n,mn,mn1) *(11.7+0.51*d(n+1,mn1)**1.38))))=e=cost;*

The model was run and resulted in the pipe properties seen in Table 2 with an objective value of $67,388. *QP* and *D* are listed by pipe for ease of comparison and are zero when *x* = 0 due to that conversion.

Table 2. General GAMS output for Simple Layout

Pipe	x	qp	d
1,1,1	1	10	13.93
1,1,2	1	15	15.82
2,1,1	1	05	06.96
2,1,2	0	00	00.00
2,1,3	0	00	00.00
2,2,1	0	00	00.00
2,2,2	1	05	11.04
2,2,3	1	05	06.90

There is a difference between this generalized model and the explicit model in the results of *D*. Despite the differing values, GAMS seems to be constructing the constraints the same in each case. For example, both the explicit and general models give:

*hydraulic(n3,1).. (0)*x(n1,1,1) + (0)*x(n1,1,2) + (0)*x(n2,1,1) + (0)*x(n2,2,1) + (0)*qp(n2,1) + (0)*qp(n2,2) + (0)*qp(n3,1) + (0)*d(n2,1) + (0)*d(n2,2)+ (0)*d(n3,1) =L= 50 ; (LHS = 0)*

Once again, the diameters were verified using EPANET. As seen in Figure 4, the constraints are still met, though the pressure heads are unnecessarily high; this results in larger diameters and a more expensive system. The full GAMS model and readout for the simple layout can provide up on request.

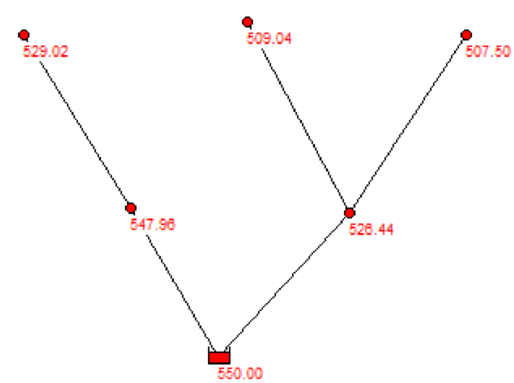

Figure 4. EPANET Pressure heads of a general simple system

2.4 Main Layout

The main scenario that this report is attempting to evaluate has five isonodal lines with up to four nodes on each line. While the demands and friction factors are the same as in the simple scenario, the spatial distribution is shown in Fig. 5.

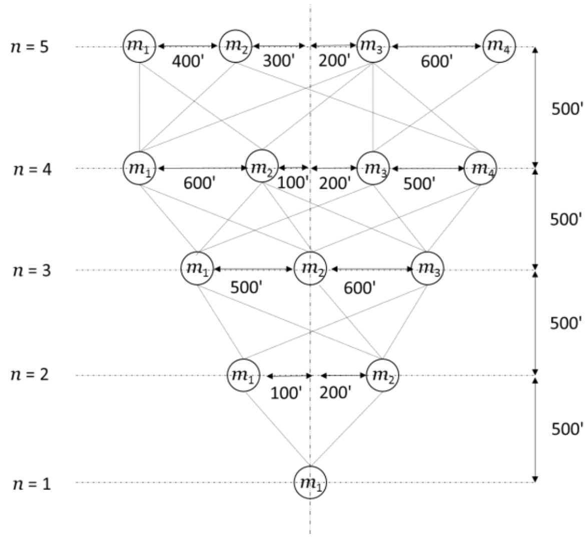

Figure 5. Main layout

The GAMS model follows the same setup as that used for the simple model. The sets and alias are adjusted to fit the larger number of isonodal lines and nodes.

```
n  5 isonodal lines /n1, n2, n3, n4, n5/
mn 4 possible nodes on n /1, 2, 3, 4/;
alias (mn, mn1, mnm1, mnm2, mnm3, mnm4);
```

The connectivity, flow, and cost equations require no adjustments. The hydraulic constraints, due to the additional number of isonodal lines to trace back across, must be expanded to include these new summations.

```
hydraulic(n,mn)..h(n,mn)=l=h('n1','1')-coeffj*
(sum(mnm1,(l(n-1,mnm1,mn)*x(n
1,mnm1,mn)*qp(n,mn)**2/((d(n,mn)/12)**5) )$(ord(n)>1)
+(sum(mnm2,
(l(n-2,mnm2,mnm1)*x(n-2,mnm2,mnm1)*qp(n-1,mnm1)**2/((d(n-
1,mnm1)/12)**5) )$(ord(n)>2) +(sum(mnm3,
(l(n-3,mnm3,mnm2)*x(n-3,mnm3,mnm2)*qp(n-2,mnm2)**2/((d(n-
2,mnm2)/12)**5))$(ord(n)>3)+sum(mnm4,
(l(n-4,mnm4,mnm3)*x(n-4,mnm4,mnm3)*qp(n-3,mnm3)**2/((d(n-
3,mnm3)/12)**5))$(ord(n)>4)))))));
```

While not covered in the bulk of this report, the GAMS model for the simple layout included a conversion from nodal notation of *QP* and *D* to a pipe identification. However, the GAMS demo limit was exceeded, so these purely clarifying equations were removed for the main layout. The existing pipes are listed in the Table 3. The objective value was $379,511. The values were entered into EPANET to check the hydraulics as seen in Figure 6. Unfortunately, the model is once again producing heads that are unnecessarily high and, hence, expensive. The full GAMS model and readout for the main layout can be provided up on requested.

Explicitly writing out the equations for the simple layout shows that the constraints are correct based on the engineering of the problem. EPANET was used to validate that the determined values were all acceptable.

The generalized models worked correctly in terms of flow and connectivity. However, the output diameters are locally optimized as needed. While the demand flows and pressures are still met, this results in a optimum capital cost. GAMS appears to be constructing the hydraulics constraints correctly as they are identical to those constructed in the written-out version. In addition, the model would be improved by putting constraints on pipe diameters to match what is commercially available in the area of application.

Table 3.General GAMS output for Main Layout

Pipe	x	qp	d
1,1,1	1	10	29.28
1,1,2	1	55	49.97
2,1,1	1	5	18.96
2,2,2	1	25	31.40
2,2,3	1	25	31.40
3,2,1	1	15	21.53
3,2,2	1	5	15.26
3,3,3	1	15	21.53
3,3,4	1	5	15.26
4,1,1	1	5	11.81
4,1,2	1	5	12.40
4,3,3	1	5	11.81
4,3,4	1	5	12.91

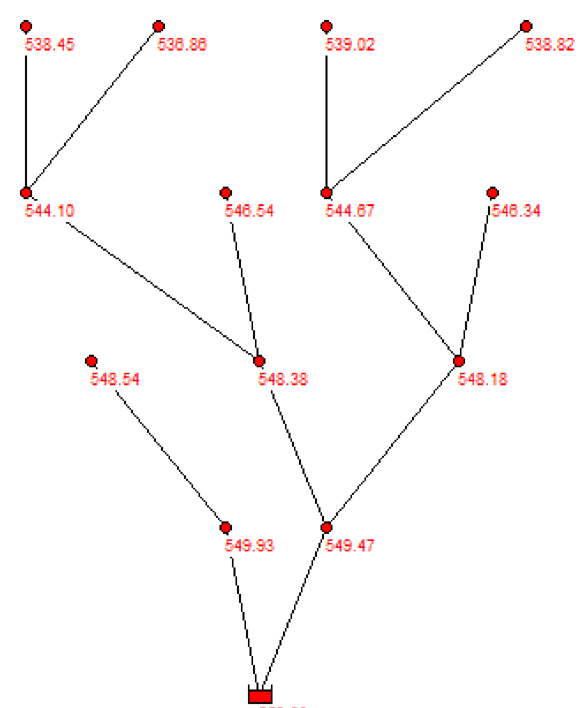

Figure 6. EPANET pressure heads of general main layout (to scale)

Declaration of competing interest

The authors declare no conflicts of interest.

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