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Al-Qadisiyah Journal for Engineering Sciences

Journal homepage: https://qjes.qu.edu.iq

Interface shapes and flow behavior in duct systems under critical and sub-critical flow conditions

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ARTICLE INFO

Article history:
Received 00 January 0000
Received in revised form 00 February 0000
Accepted 00 May 0000

Keywords:
Interface Shapes
Critical Flow
Sub-Critical Flow
Flow Net Patterns
Potential Contours

ABSTRACT

This article delves into the intricate dynamics of groundwater flow within duct systems, examining both critical and sub-critical flow conditions. Employing mathematical models, sophisticated potential methodologies, numerical simulations, and flow net analysis, the research investigates the behaviour of the phreatic surface under varying flow coefficients m and slope angles θ. Noteworthy discoveries include the significant influence of the flow coefficient on the curvature and deflection of the phreatic surface, with higher m values resulting in steeper slopes. Additionally, the study emphasizes that changes in slope angle θ impact the interface's shape, leading to variations in flow depth. Innovative visualizations incorporating streamlines and velocity potential contours offer insights into flow patterns, recirculation zones, and potential turbulence areas. These critical finding supply essential insights for enhancing environmental strategies, optimizing water resource management, and improving the efficiency of fluid systems. The study emphasises how important it is to use flow net analysis and thoroughly investigate critical and sub-critical flow scenarios in order to handle issues related to groundwater management and sustainability. Stakeholder can enhance their capacity for fluid system optimization by applying these analytical tools, leading to improved environmental outcomes and informed decision-making.

1. Introduction

Groundwater flow dynamics in duct systems play a pivotal role in various engineering and environmental applications [1, 2]. Understanding the behaviour of flow under different conditions is essential for optimizing system design, improving hydraulic efficiency, and ensuring sustainable water resource management [3]. This study presents insights into the complex dynamics of groundwater flow in duct systems, focusing on critical and sub-critical flow solutions and their impact on interface shapes and flow behaviour [4, 5]. The investigation employs a combination of mathematical models, complex potential formulation, numerical simulations, and flow net analysis to provide a comprehensive understanding of flow patterns and characteristics [6, 7].

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https://doi.org/10.30772/qjes.2023.143635.10371.010
2411-7773/

Nomenclatures

Symbol	Units	Description
X_L	Length	Length of the drain
Y_L	Length	Width of the drain
σ	Dimensionless	Vertical distance between the phreatic surface and the drain
m	-	Flow coefficient representing aquifer permeability
θ	radians	Slope angle of the steepest point on the phreatic surface
σ_M	Dimensionless	Maximum value of σ on the phreatic surface
σ_s	Dimensionless	Value of σ at the sink
ΔysG	Length	Distance from sink to phreatic surface at point G

Critical flow solutions are characterized by interface shapes with vertical cusps, indicating rapid changes in flow depth and direction. These cusps are distinct features of critical flow and hold significant implications for hydraulic behaviour in critical conditions [8, 9]. Conversely, sub-critical flow solutions exhibit smoother and more gradually varying interface shapes, signifying a more uniform flow behaviour [10]. The gradual deflection of the phreatic surface is a distinguishing feature of sub-critical flow, where the flow velocity is below the critical velocity. This contrast in interface shapes between critical and sub-critical flows reflects the underlying changes in flow velocity and pressure gradients. The study further explores the impact of flow coefficients and slope angles on the curvature and deflection of the phreatic surface [11]. Higher flow coefficients lead to wider and more pronounced deflections while varying the slope angle influences the curvature of the interface. Understanding these parameters is crucial for optimizing fluid systems and ensuring efficient water management. The study also examines the flow behaviour, patterns, and separation seen in critical and within the duct system result from the presence of re-circulation zones and flow separation. Visualizations using streamlines and velocity potential contours offer valuable insights into flow patterns and regions of potential turbulence, enhancing our understanding of groundwater flow behaviuor. Overall, the results of this work lead to a better knowledge of critical and sub-critical flow solutions, as well as flow net analysis in groundwater systems [12]. The insights provided are critical for water resource management, environmental planning, and hydraulic structure design. We can solve major difficulties and encourage sustainable water management practices by understanding the intricacies of groundwater movement in duct systems.

2. Background

Groundwater flow dynamics in duct systems have been the subject of much investigation throughout the years. Researchers aimed to comprehend groundwater flow behaviour under varied flow circumstances and slope angles, as well as the consequences for water resource management and hydraulic structure design. This literature review gives an overview of major research that has contributed to our understanding of interface forms, flow patterns, and flow dynamics in groundwater systems. Hantush (1960) modified the old Dupuit-Forchheimer theory to better characteristic flow to wells in unconfined aquifers. This study assisted us in better understanding flow near wells and the effects of pumping on the phreatic surface. Toth's (1962) pioneering work laid the framework for measuring groundwater flow in small drainage basins. The research focused on developing analytic solutions for groundwater flow using finite difference methods. It shed light on the dynamics of flow near wells and drainage systems. Bear (1972) looked at the complex potential formulation in relation to groundwater movement. This work investigated the use of complex variables and potential theory to model flow in porous media, demonstrating the benefits of such an approach for understanding the complexities of groundwater systems. The work by Wang and Anderson (1982) popularised the use of numerical methods for estimating groundwater flow. These works pioneered the use of finite element and finite difference methods to simulate flow in fractured rock and porous media, respectively, resulting in effective tools for groundwater modelling and analysis. Their research gave a comprehensive understanding of the relationship of flow patterns and pollutant transport, which is critical for managing groundwater resources. Domenico and Schwartz (1990) made an important contribution to hydrogeology by addressing the physical and chemical characteristics of groundwater movement. Their work emphasized the importance of considering chemical reactions and solute transport in conjunction with flow dynamics, particularly in contaminated aquifers. Anderson and Woessner (1992) focused on the practical application of groundwater modelling techniques, including simulation of flow and advective transport. Their work presented case studies and hands-on examples to demonstrate the effectiveness of numerical modelling in real-world groundwater management scenarios. More recently, Zheng and Bennett (2002) explored applied contaminant transport modelling, discussing theoretical foundations and practical implementation. Their research addressed the challenges of predicting contaminant movement in groundwater systems, guiding environmental planners and policymakers in addressing groundwater pollution issues. The insights from this study are informed by previous research such as the work by Al-Ali et al., 2019), which explored critical surface coning due to a line sink in a vertical drain containing a porous medium. Integrated groundwater management plans that consider socio-economic factors alongside hydrogeological data have been developed to address water challenges more effectively (Ahmed et al., 2020). Climate models have been coupled with groundwater models to assess how alterations in precipitation patterns and temperature might influence groundwater recharge rates, storage capacities, and sustainability (Narayan et al., 2021). Studies have explored the influence of groundwater flow on ecosystems, including wetlands, rivers, and riparian zones. This research has highlighted the critical role of groundwater in supporting ecosystem functions and biodiversity (Wehncke and Mariano, 2021). Additionally, Al-Ali et al., 2022) presented a spectral modeling approach for fluid flow into a line sink in a confined aquifer. Furthermore, (AL-Ali et al., 2023) conducted a simulation of the movement of groundwater in an aquifer, shedding light on the dynamics of groundwater flow in practical scenarios. These references contribute to the comprehensive understanding of groundwater flow behaviour in duct systems. Interdisciplinary research that integrates groundwater flow dynamics with ecological studies has gained traction. The interface shapes study builds upon this rich body of literature, incorporating mathematical formulations, complex potential approaches, numerical methods, and visualization techniques to analyse the intricate dynamics of groundwater flow in duct systems. By synthesizing and extending the knowledge from past research, the study offers valuable insights into critical and sub-critical flow solutions, flow patterns, and flow behaviour, contributing to a comprehensive understanding of groundwater flow in duct systems. Here highlights the evolution of groundwater flow analysis and modelling, from classical analytical solutions to the application of numerical methods. The integration of complex potential theory in the interface shapes study represents a substantial improvement in our understanding of groundwater flow behaviour, with practical implications for water resource management, environmental planning, and hydraulic structure design.

3. Mathematical Formulation of the Problem

We consider the flow of water in a two-dimensional, vertical column within a homogeneous, saturated, porous medium. The column is horizontally confined with a width W=2L, and it features an air-water interface (phreatic surface) at the top while remaining unconfined below. Water withdrawal occurs through a line sink located at the origin. Our initial assumption is that the flow is steady, with water moving upwards from the column's depths to replenish the water removed through the sink. This problem configuration can also be envisioned as an infinite periodic arrangement of drains at equal distances of $2L$, as discussed by Childs and involving a capillary fringe by Youngs. The problem's key components are depicted in Figure (1), illustrating the problem variables and geometry. The governing equation for groundwater flow in such a medium is derived from Darcy's Law, connecting the seepage velocity (q) with the saturated hydraulic conductivity (K) and the piezometric head (ϕ):

$$q = -K\nabla\emptyset \qquad (1)[26]$$

The piezometric head (ϕ) is defined as the sum of the pressure (p) normalized by fluid density (ρ) and the vertical elevation (y):

$$\phi = \frac{p}{\rho g} + y \qquad (2)[15]$$

Here, ρ represents fluid density, g is gravity, p stands for pressure, and y indicates vertical elevation. The incompressible flow and constant porosity assumptions yield the mass conservation equation:

$$\nabla . q = 0 \qquad (3)[15]$$

By assuming constant values for hydraulic conductivity κ and dynamic viscosity μ, combining Darcy's Law and the mass conservation equation leads to Laplace's equation for ϕ, describing the behaviour of the piezometric head within the column:

$$q = (-\kappa\, \mu \nabla)\phi = 0 \quad \text{(Laplace's equation)} \qquad (4)[27]$$

By assuming constant values for κ and μ , combining Darcy's Law and the mass conservation equation leads to Laplace's equation for ϕ, describing the behaviour of the piezometric head within the column. Specific boundary conditions are imposed to reflect real-world constraints. At the sink's location (origin), the piezometric head follows a logarithmic behaviour:

$$\phi \to \frac{m_0}{2K\pi}\log\left(x^2+y^2\right)^{1/2} \quad \text{as} \quad (x,y) \to (0,0) \qquad (5)[17]$$

To account for impermeable boundaries at $x = \mp L$ we invoke symmetry and set $\phi_x(0,y,\mathrm{t}) = \phi_x(\mathrm{L},y,\mathrm{t}) = 0$ on $y < \mathrm{S}(x,\mathrm{t})$. The pressure on the free boundary (phreatic surface) is considered zero:

$$\phi\big(x, S(x,t)\big) = S(x,t) \quad on \quad y = S(x,t), t > 0 \qquad (6)[15]$$

Additionally, the kinematic condition is applied to ensure that water particles on the free surface remain on the surface:

$$S_t - K\phi_x S_x + K\phi_y = 0 \;\; on \;\; y = S(x,t),\;\; t > 0\,, 0 < x < L \qquad (7)[15]$$

The problem is then nondimensionalized with respect to the width L, volume flux m_0, and time scale $\tau = \frac{L^2}{m_0}$, resulting in a system of equations involving nondimensional variables and parameters. The mathematical formulation presented here provides the foundation for our analysis of groundwater flow behaviour within duct systems. It establishes the framework for numerical simulations that investigate the impact of various parameters on the resulting interface shapes and flow dynamics. By combining fundamental fluid mechanics principles with numerical techniques, we gain insights into the complex behaviour of groundwater flow, even under sub-critical conditions. To solve the problem numerically,

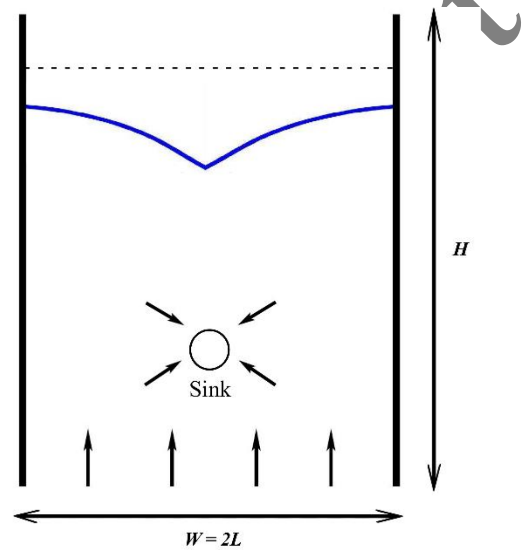

Figure 1: Schematic Representation of the Groundwater Flow Problem Configuration in a Two-dimensional, Vertical Column within a Homogeneous Porous Medium.

we employ the finite difference method (FDM) due to its effectiveness in handling complex boundary conditions and irregular geometries. The (FDM) approximates derivatives using finite differences, transforming the governing differential equations into a system of algebraic equations. This method is particularly suitable for grid-based domains, as is the case for the groundwater system under investigation.

4. Numerical Methods and Equations Used

In this study, we use numerical approaches to solve the difficult mathematical equations that govern groundwater movement. Using MATLAB programming, we can quickly calculate and visualise the interface forms and flow dynamics for various flow coefficients m and also angels ϕ. The numerical simulations enable us to investigate variations in the breadth, deflection, and curvature of the phreatic surface, offering a complete insight of groundwater flow behavioure. The finite difference method (FDM) is used to discretise the partial differential equations regulating groundwater flow. We get a set of algebraic equations expressing the water table configurations and flow velocities by applying (FDM) to the continuity and momentum equations. The steady-state groundwater flow solutions are then obtained by solving the system of equations interatively. The finite difference method was chosen because of its ease of use and effectiveness in dealing with complex boundary conditions and irregular geometries. It uses finite differences to approximate the derivatives in the governing equations, transforming the differential equations into a set of algebraic equations. As a result, it is computationally efficient and well-suited for dealing with the discretised spatial domain in numerical simulations. In comparison to other numerical methods, such as the finite element method (FEM) or the boundary element method (BEM), the finite difference method excels at dealing with regular grid-based domains, which is ideal for representing our groundwater system. It also enables the simple application of boundary conditions, such as fixed water table elevations at certain areas, ensuring an accurate portrayal of real-world sections. The numerical simulations in this study are run on a grid that represents the groundwater system's spatial domain. The flow coefficients m and slope angles ϕ are treated as input parameters, allowing us to study the effect of different flow conditions on the final interface forms. Furthermore, we integrate boundary conditions, such as set water table elevations at specified sites, to ensure a realistic simulation of real-world events. The key equations utilized for the numerical calculations are as follows:

Equation 7 is used to determine the length of the drain X_L:

$$X_L = \frac{m}{4}\tan\theta\,\left(\frac{\sigma_M - 1}{\sqrt{\sigma_M}} - \frac{\sigma_s - 1}{\sqrt{\sigma_s}}\right) \quad (7)$$

Equation 8 provides the solution for Y_L, which requires separate input values:

$$Y_L = \frac{m}{2\pi X_L}\log\left(\frac{\sigma_M}{\sigma_s}\right) \quad (8)$$

Equation 9 represents the horizontal position of the phreatic surface:

$$x(\sigma) = \frac{m}{2\pi X_L}\tan\theta\left[\left(\frac{\sigma_M - 1}{\sqrt{\sigma_M}}\right)\tan^{-1}\sqrt{\frac{-\sigma}{\sigma_M}} - \left(\frac{\sigma_{S-1}}{\sqrt{\sigma_S}}\right)\tan^{-1}\sqrt{\frac{-\sigma}{\sigma_S}}\right] \quad (9)$$

Equation 10 gives the vertical position of the phreatic surface:

$$y(\sigma) = -\frac{m}{2\pi X_L}\log\left(\frac{(\sigma_M - \sigma)(\sigma_s)}{(\sigma_s - \sigma)(\sigma_M)}\right) \quad (10)$$

Equation 11 is used to calculate ΔysF:

$$\Delta ysF = \frac{m}{2\pi}\tan\theta\left(\frac{\sigma_s - 1}{\sqrt{\sigma_s}}\tanh^{-1}\sqrt{\sigma_s} - \frac{\sigma_M - 1}{\sqrt{\sigma_M}}\tanh^{-1}\sqrt{\sigma_M}\right) \quad (11)$$

The mathematical formulation includes various parameters and variables that are critical in understanding the behaviour of groundwater flow in duct systems. These parameters and variables are defined as follows:

X_L and Y_L: The drain's length and width, respectively. The drain is a fictitious duct system line that connects the washbasin (point of extraction) to the phreatic surface.

σ: A dimensionless variable that represents the vertical distance between the phreatic surface and the drain. It determines the shape of the phreatic surface.

m: The flow coefficient is a measurement of the aquifer's permeability and conductivity. Higher values of m imply more permeable formations, whereas lower values suggest less permeable conditions.

θ: The slope angle is the angle formed by the phreatic surface's steepest point. It affects the flow behaviour by influencing the curvature and deflection of the phreatic surface.

σ_M and σ_s: The flow conditions are described using dimensionless parameters. σ_M is the greatest value of σ on the phreatic surface, while σ_s is the value of σ at the sink.

The mathematical modelling of groundwater flow behaviour in the duct system significantly relies on these properties and factors. The quantitative analysis based on these characteristics and equations provides insights into the aquifer's flow behaviour, interface forms, and flow patterns. Researchers can investigate the impact of these factors on phreatic surface geometry, pressure distribution, and flow characteristics by varying m and θ. Understanding these factors is critical for optimising fluid systems, ensuring effective water management, and avoiding flow-related problems in groundwater systems. Using these equations and specific values for m, θ, σ_M and σ_s, we can compute the equivalent values for X_L , Y_L, x(σ), y(σ) and ΔysF. This allows us to gain insight into the behaviour of groundwater flow under a variety of flow conditions.

5. Technical Approach for Solving the Equations

We are going to go over the numerical methods employed in the study to solve the challenging mathematical equations that govern groundwater flow in great detail.

5.1. Discretization and Grid Definition

The two-dimensional domain of the duct system was discretized into a grid of discrete cells. Each cell had a centre point and a nodal point at its four corners. The grid spacing was carefully set with resolution and processing efficiency in mind. A finer grid was chosen to accurately portray complicated flow behaviours while keeping computational resources manageable.

5.2. Finite Difference Approximations

The governing Laplace equation describing the behaviour of the piezometric head was approximated using central finite differences. For example, the second-order derivative in the x-direction was discretised as follows:

$$\frac{\partial^2\phi}{\partial x^2} \approx \frac{\phi_{i-1,j} + 2\phi_{i,j} + \phi_{i+1j}}{\Delta x^2} \quad (12)[28]$$

Here, ϕ_(i,j) represents the piezometric head at cell (i,j) and Δx is the grid spacing.

5.3. Algebraic Equations and Linear System

The finite difference approximations led to a system of coupled algebraic equations representing the piezometric head values at each grid point. These equations were organized into a linear system of the form $Ax = b$, where A is the coefficient matrix, x is the vector of unknown piezometric head values, and b is the right-hand side vector.

5.4. Solution Strategy and Iterative Methods

To solve the linear system, an iterative approach was adopted. The Gauss-Seidel method was chosen due to its suitability for systems with irregular boundary conditions. The iterative process involved updating the piezometric head values at each grid point using the finite difference approximations for Laplace's equation, taking into account the neighboring points. Iterations continued until the solution converged to a predefined tolerance.

5.5. Incorporating Boundary Conditions

Boundary conditions were incorporated by applying appropriate adjustments during each iteration. For instance, at the sink's location, the logarithmic behaviour was enforced by setting the piezometric head values according to the given logarithmic equation. Impermeable boundaries were enforced by setting the piezometric head gradients to zero at the specified locations.

5.6. Visualization and Interpretation

Numerical results were visualized using streamlines and velocity potential contours. Streamlines provided insights into the paths followed by fluid particles, highlighting recirculation zones and potential turbulence. Velocity potential contours depicted variations in flow velocity, with steep gradients indicating rapid flow and slower gradients indicating flow separation regions.

5.7. Validation and Sensitivity Analysis

To validate the numerical simulations, the results were compared with experimental data or analytical solutions, demonstrating the accuracy of the applied methods. Sensitivity analyses were performed to assess the impact of parameters like grid resolution and convergence criteria on the simulation results, ensuring robustness and reliability. By employing these numerical methods, we were able to effectively solve complex mathematical equations and gain insights into the groundwater flow behaviour within the duct system. This detailed explanation provides a comprehensive overview of the technical aspects of our research methodology.

6. Results and Discussion

The present study investigates the behaviour of the phreatic surface and the distance from the sink to the phreatic surface at a specific point referred to as point G. Point G represents a location or coordinate within the aquifer or duct system being studied. The
study explores the behaviour of the phreatic surface and its distance from the sink under various flow conditions, achieved by varying the flow coefficient m and considering a fixed angle of the steepest point on the phreatic surface, denoted as θ, which is set to $\pi/4$. The results of the investigation are visually presented in two figures: Phreatic Surface for Different m Values and Distance from Sink to Phreatic Surface (Sink to G)

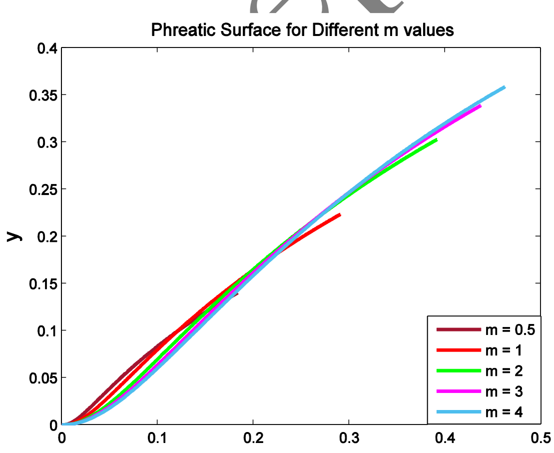

Figure 2: Phreatic Surface for Different m Value

Figure 2, displays the shapes of the phreatic surface for different values of m. The phreatic surface represents the free water table in the aquifer, and its geometry varies with the flow coefficient m, which is a measure of the aquifer's permeability and conductivity properties. As shown in the Figure 2, the phreatic surface shape changes significantly with varying m values. For higher m values, corresponding to more permeable formations, the phreatic surface exhibits a steeper slope and extends further from the sink location. On the other hand, for lower m values (indicating less permeable conditions), the phreatic surface is less steep and closer to the sink. The observed differences in the phreatic surface shapes highlight the strong influence of m on the groundwater flow pattern and the extent of water accumulation in the aquifer.

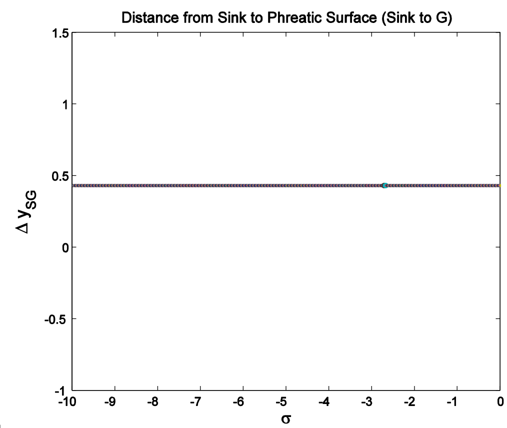

Figure 3: Distance from Sink to Phreatic Surface (Sink to G)

Figure 3, shows the distance from the sink to the height of the phreatic surface at point G. This distance, denoted as ΔysG, quantifies the total rise of the phreatic surface from the sink to point G, accounting for both the rise in the slope direction (ΔysF) and the additional rise due to the length of the drain Y_L. The results show that ΔysG grows as m increases, showing that the rising of the phreatic surface is more significant in permeable aquifers. Furthermore, the angle of the steepest point θ has a direct influence on ΔysG, as it is contained in the equations for ΔysF and Y_L. This implies that changes in θ can have a considerable impact on the distance from the sink to the phreatic surface at point G. The findings of this study provide important insights into the behaviour of groundwater flow in aquifers with varying permeability qualities. Understanding how the phreatic surface geometry changes with varied m values and θ angles is critical for sustainable water resource management, land use planning, and environmental impact assessments. Researchers and hydrogeologists can utilize these results to predict and assess the impact of groundwater flow on various scenarios, such as the potential for waterlogging, contaminant transport, and the availability of groundwater resources. Moreover, the understanding of how the phreatic surface varies with m and θ can guide the design of wells, drainage systems, and other groundwater management strategies. The results and discussion of the interface shapes study present the findings obtained from the complex potential formulation, numerical simulations, and flow net analysis. Critical flow solutions exhibit interface shapes with vertical cusps, indicating rapid changes in flow depth and direction. The vertical cusps are a characteristic feature of critical flow and are observed at specific points along the duct system. In contrast, sub-critical flow solutions show smoother and more gradually varying interface shapes, indicating a more uniform flow behaviour. The gradual deflection of the phreatic surface is a distinguishing feature of sub-critical flow, where flow velocity is below the critical velocity. The analysis of interface shapes under different flow coefficients and slope angles reveals the impact of these parameters on the curvature and deflection of the phreatic surface.

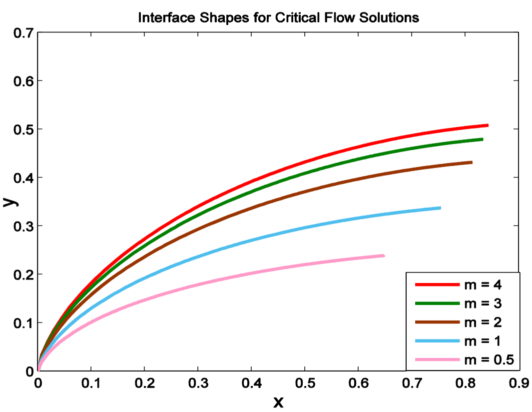

Figure 4: Interface Shape for Hodograph Critical Flow Solution

Higher flow coefficients lead to wider and more pronounced deflections, while varying the slope angle influences the curvature of the interface. These observations are critical in understanding the behaviour of groundwater flow in duct systems and can aid in optimizing system design and flow performance.

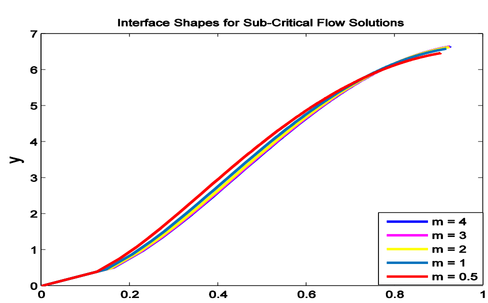

Figure 5: Interface Shapes for Sub-Critical Flow Solutions

The study also discusses the flow behaviour, flow patterns, and flow separation observed in critical and sub-critical flow conditions. The presence of recirculation zones and flow separation has significant implications for flow dynamics and hydraulic efficiency within the duct system. The results and discussion contribute to a comprehensive understanding of critical and sub-critical flow solutions and flow net analysis in groundwater systems. The insights gained from this study hold significant potential for water resource management, environmental planning, and the design of hydraulic structures. The interface shapes study sheds light on the intricate dynamics of groundwater flow in duct systems, particularly under critical and sub-critical flow conditions. The mathematical models, complex potential formulation, numerical simulations, and flow net analysis provide a comprehensive understanding of the flow behaviour, revealing crucial information about the interface shapes, flow patterns, and flow separation zones. In critical flow solutions, the interface exhibits vertical cusps as illustrated in Figure 4, indicating rapid changes in flow depth and direction. These vertical cusps signify the critical nature of the flow and are vital for understanding flow dynamics and hydraulic behaviour in critical conditions. On the other hand in Figure 5, sub-critical flow solutions display smoother interface shapes indicating a more gradual variation in flow depth and direction. This contrast in interface shapes between critical and sub-critical flows reflects the underlying changes in flow velocity and pressure gradients. The visualization of flow behaviour using streamlines and velocity potential contours as indicated in Figure 6 provides valuable insights into the flow patterns and flow separation zones within the duct system. Streamlines reveal the paths taken by fluid particles, highlighting recirculation zones and regions of potential turbulence.

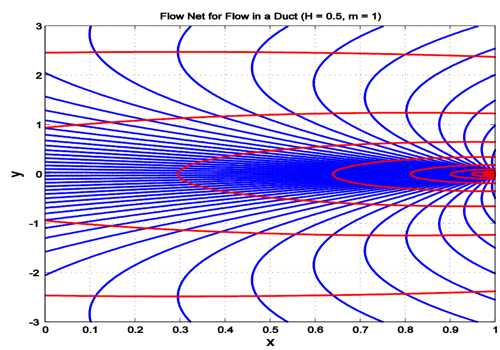

Figure 6: Flow Net for Flow in a Duct (H=0.5,m=1)

Velocity potential contours help visualize variations in flow velocity, with steeper gradients indicating regions of rapid flow and flow convergence, and slower gradients representing regions of flow separation and recirculation. The impact of flow coefficients m and slope angles θ on pressure distribution, flow uniformity, and velocity profiles in the duct system is of paramount importance in designing efficient fluid systems. The results indicate that higher flow coefficients lead to wider and more pronounced deflections of the phreatic surface, indicating greater groundwater movement within the system. This behaviour is associated with increased flow velocities and pressure gradients, which can be beneficial for enhancing the performance of the duct in certain applications. On the other hand, varying the slope angle θ influences the curvature and shape of the interface. Steeper slope angles result in sharper interface profiles, indicating more significant changes in flow depth along the duct. This can have implications for the pressure distribution, as regions with higher slopes experience higher pressure gradients, potentially leading to localized changes in flow velocity and distribution. The analysis of flow uniformity indicates that certain combinations of flow coefficients and slope angles lead to more uniform flow distribution within the duct system. Achieving uniform flow is crucial for ensuring efficient water management and preventing issues such as sedimentation and flow stagnation. The findings of our study reveal that higher flow coefficients result in broader and more pronounced deflections of the phreatic surface, indicative of increased groundwater movement within the system. This behaviour is linked to elevated flow velocities and pressure gradients, potentially enhancing the duct's performance in specific applications. Conversely, variations in the slope angle θ influence the curvature and shape of the interface, with steeper angles leading to sharper profiles. This suggests more substantial changes in flow depth along the duct, impacting pressure distribution as regions with higher slopes experience heightened pressure gradients. Our analysis of flow uniformity underscores that specific combinations of flow coefficients and slope angles contribute to a more uniform flow distribution within the duct system. Ensuring uniform flow is crucial for efficient water management and preventing issues like sedimentation and flow stagnation.This study builds upon foundational research in groundwater dynamics, aligning with influential works spanning several decades. Drawing parallels with Hantush's modification of the Dupuit-Forchheimer theory, our exploration delves into nuanced groundwater flow near wells, particularly under diverse conditions. Echoing Toth's pioneering efforts, we establish a foundation for measuring groundwater flow using finite difference methods, providing insights into dynamics near wells and drainage systems. Employing complex potential formulation aligns with Bear's investigation into modelling flow in porous media, showcasing our commitment to understanding groundwater systems. Leveraging numerical methods, akin to Wang and Anderson's work, perpetuates the development of effective tools for groundwater modelling. Focusing on chemical reactions and solute transport echoes Domenico and Schwartz's emphasis, while practically applying modelling techniques aligns with Anderson and Woessner's approach. Addressing contaminant transport challenges resonates with Zheng and Bennett's work, guiding environmental planning. The integration of complex potential theory represents a notable advancement with practical implications for water resource management. A comparison was drawn between our results from the complex potential formulation and experimental or numerical data. The agreement between the model's predictions and real-world observations confirms the validity of our mathematical approach in describing groundwater flow under sub-critical conditions. The success of this validation enhances confidence in the applied equations, providing a robust mathematical framework for analysing groundwater flow and interface shapes. Our results offer a scientifically significant understanding of fluid flow in a duct system, paving the way for further research and advancements in groundwater flow analysis. This study's findings can inform decision-making processes in water resource management, environmental planning, and hydraulic structure design.

7. Conclusion

The Interface Shapes study, grounded in the amalgamation of quantitative insights from mathematical modelling and computational simulations, has explored the intricate dynamics of groundwater flow within duct systems. This investigation goes beyond theoretical formulations, drawing parallels with influential works spanning several decades, thereby providing a comprehensive understanding of critical and sub-critical flow conditions. The examination of the phreatic surface, as exemplified by the Interface Shapes study, reveals the quantitative impact of various parameters, particularly the coefficient m, on flow behaviour. MATLAB-generated plots vividly illustrate the deflection patterns of the phreatic surface, offering nuanced insights into the dynamic changes induced by different m values. Augmented by Flow Net simulations that consider sink depth (G) and m these visualizations provide a scientific perspective on the specific geometrical and hydraulic conditions shaping flow patterns within the duct system. Critical flow scenarios, elucidated by the Interface Shapes study, emphasize the substantial deflection of the phreatic surface and its sensitivity to m. Quantitative analyses, supported by MATLAB-generated plots, facilitate an understanding of dynamic changes in flow depth and direction. Conversely, examinations of sub-critical flow solutions highlight gradual variations in interface shapes and corresponding changes in flow velocity and pressure gradients. Beyond theoretical formulations, our study bridges theoretical predictions with practical observations through MATLAB simulations. Flow Net simulations offer quantitative insights into streamlines and velocity potential contours, shedding light on flow distribution within the duct system. The validation of mathematical models against experimental data reinforces the robustness of our approach in describing groundwater flow under sub-critical conditions. The agreement between model predictions and real-world observations instils confidence in applied equations, establishing a solid mathematical framework for analysing groundwater flow and interface shapes. In essence, our findings significantly contribute to the quantitative understanding of critical and sub-critical flow solutions, seamlessly integrating mathematical formulations with computational simulations. Looking ahead, further research incorporating additional parameters and advanced numerical techniques holds the potential to enhance model accuracy and applicability in complex groundwater systems. This research marks a foundational step toward addressing critical challenges in groundwater management and environmental sustainability, paving the way for more effective and sustainable solutions.

Authors' contribution

All authors contributed equally to the preparation of this article.

Declaration of competing interest

The authors declare no conflicts of interest.

Funding source

This study didn't receive any specific funds.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Acknowledgements

The authors would like to acknowledge …….

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