Contents lists available at http://qu.edu.iq

Al-Qadisiyah Journal for Engineering Sciences

Journal homepage: http://qu.edu.iq/journaleng/index.php/JQES

A graph-based method for detecting isomorphism of geared kinematic chains using a circuit matrix and vertex degree array.

Sajad Abdali [*], Farah Saoud, and Essam Esmail

Department of Mechanical Engineering., College of Engineering, University of Al-Qadisiyah, Iraq

ARTICLE INFO

Article history:
Received 00 January 0000
Received in revised form 00 February 0000
Accepted 00 May 0000

Keywords:
Circuit Matrix
Isomorphism
kinematic chains
Spanning tree
Planetary gear mechanism (PGM)

ABSTRACT

Several approaches for detecting isomorphism in kinematic chains have been developed in recent literature. If two kinematic chains have a 1-1 correspondence and their incidences are maintained, they are isomorphic. In this work, a matrix-based method for identifying isomorphism is presented. The new method is based on fundamental circuits, vertex degrees, and spanning trees. A unique identifier for isomorphic graphs is proposed. Two graphs are isomorphic if their isomorphic identification numbers have the same value. This reduces the structural isomorphism test to a comparison of the isomorphic identification numbers of the two graphs under consideration. Regardless of vertex labeling of the graphs, which is problematic in other ways, similar isomorphic identification numbers are generated for isomorphic graphs. The new method is a comprehensive, systematic way for detecting isomorphism during the synthesis of kinematic chains. Isomorphic graphs are identified regardless of graph representation. The new approach is verified by the atlas of 6-link 2- degree of freedom planetary gear mechanisms (PGMs), the atlas of 5-link 2-degree of freedom planetary geared cam mechanisms (PGCMs) as well as some PGMs and PGCMs.

1. Introduction

Planetary gear train (PGT) based mechanisms are extensively employed in vehicles transmissions, robot reduction devices, pulley blocks, machine and electrical equipment, robots, etc. The creation of planetary gear mechanisms (PGMs) is a crucial issue in the development of PGT-based mechanical equipment. PGTs are composed of central gears and planetary gears that revolve around them. Each pair of meshing gears is supported by a link known as the planet gear carrier, which keeps the distance between gear centers constant. Fig. 1(a) depicts the schematic diagram of the famous Simpson planetary gear mechanism (PGM). It is a seven-link two-DOF fractionated PGT comprised of a six-link one-DOF PGT connected in series with its casing. The additional degree of mobility is achieved simply by allowing the gear train to spin as a unit. A fractionated PGM has a separation link that can be broken into two parts to separate the PGM into two distinct components. As illustrated in Fig. 2 (b), the basic structure of this mechanism is a 6-link 1-DOF non-fractionated PGT. With the topology of a mechanism described by the topological graph and incidence matrix, the mechanism synthesis can be readily handled by a computer. Then it will be possible to automate the synthesis of mechanisms. A crucial step in the structural synthesis of PGMs is isomorphism determination; the accuracy of the isomorphism determination technique directly affects the quality of the results of the structural synthesis of PGMs. When two graphs are isomorphic, there is a one-to-one correspondence between each of the vertices and edges, preserving incidence.

Detecting isomorphism in kinematic chains is a complex topic that has been studied for many years. Detecting isomorphism in kinematic chains presents several challenges:

* *Corresponding author.*
E-mail address: eng.mech.4st-11@qu.edu.iq (Sajad Abdali)

https://doi.org/10.30772/qjes.2023.142060.1010
2411-7773/

Nomenclature:

C_v	The vertex-circuit matrix
$[C_v]_w$	The weighted vertex-circuit matrix
d_i	The degree of vertex i
d_{iw}	The weighted degree of vertex i
D	The vertex degree array $= [d_0, d_1, \dots, d_{v-1}]$
D_w	The weighted vertex degree array $= [d_{0w}, d_{1w}, \dots, d_{(v-1)w}]$
e	Number of edges in the graph
e_g	The number of geared edges
e_r	The number of revolute edges
F	The number of degrees of freedom.
$FCAA$	The fundamental circuit assortment array
IIN	Isomorphic identification number
v	Number of vertices in the graph

Abbreviations

PGM	Planetary gear mechanism
PGCM	Planetary geared cam mechanism
PGT	Planetary gear train
DOF	Degrees of freedom
FC	Fundamental circuit
VDS	vertex degree string

1. Complexity: Kinematic chains may be complex, making it challenging to dentify isomorphism in them.
2. Graph theory: Numerous techniques for detecting isomorphism in kinematic chains rely on graph theory, which may contain problems with representing mechanisms uniquely.
3. Computational complexity: Detecting isomorphism in kinematic chains may be computationally difficult, especially for large and intricate kinematic chains.
4. Lack of standardization: Since there is no standard method for locatingisomorphism in kinematic chains, comparing various approaches and results can be difficult.

To develop a reliable and efficient method for detecting isomorphism in kinematic chains, it is necessary to address these issues. The goal of this work is to establish a simple, entirely automatic isomorphism determination method that is based on the rooted graph and its circuit matrix.

1.1 Graph Representation

According to the method developed by Buchsbaum and Freudenstein [1], the conventional graph of the Simpson PGT can be represented, as shown in Fig. 1. (c). In graph representation, a vertex denotes a link and an edge denotes a joint. In this work, a revolute pair is represented by a thin edge, a cam pair by a thick edge, and a geared pair by a dashed edge. The conventional graph representation may lead to the generation of pseudo-isomorphic graphs [2-5]. Pseudoisomorphic graphs are those that are kinematically and functionally equivalent to their corresponding PGTs but are mathematically non-isomorphic. These PGTs are regarded as isomorphic from a functional point of view [6]. The detection of isomorphism will be greatly complicated or prone to errors if there are pseudo-isomorphic graphs present. As a result, whenever possible, such graphs should be avoided.

Yang et al. [2] introduced a new graph model for representing the structure of PGTs, which had both solid and hollow vertices. Revolute pairs with the same level are equivalent to multiple-joint and are represented by a hollow vertex. Therefore, PGTs with revolute pairs of different levels only have solid vertices. For this graph representation, there exist two different types of graphs; graphs with one or more hollow vertices and graphs without hollow vertex. Since the vertices represent the links, the number of vertices should not exceed the number of links. In a hollow vertex graph, the number of vertices exceeds the number of links, hence, there is no one-to-one correspondence with the PGT.

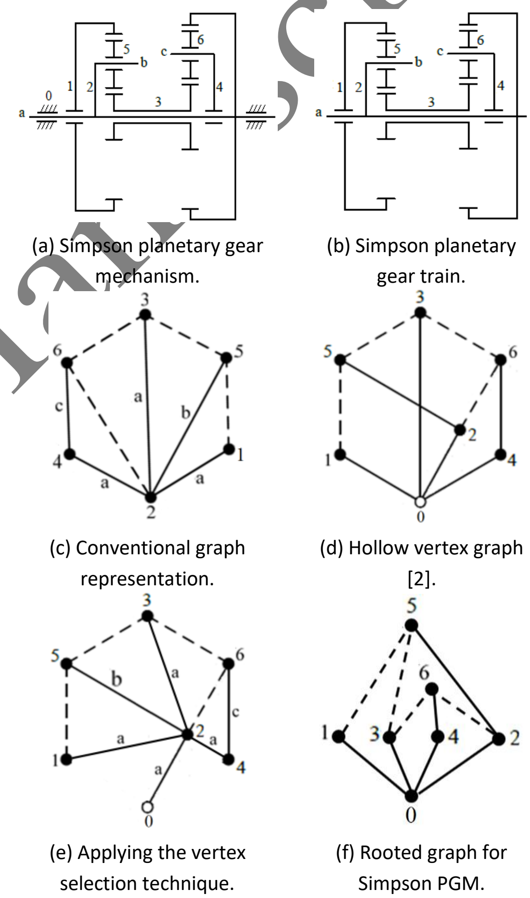


Figure 1. The Simpson gear mechanism and its conventional, hollow vertex, rooted graph representations.

1.2 Graph Representation

According to the method developed by Buchsbaum and Freudenstein [1], the conventional graph of the Simpson PGT can be represented, as shown in Fig. 1. (c). In graph representation, a vertex denotes a link and an edge denotes a joint. In this work, a revolute pair is represented by a thin edge, a cam pair by a thick edge, and a geared pair by a dashed edge. The conventional graph representation may lead to the generation of pseudo-isomorphic graphs [2-5]. Pseudoisomorphic graphs are those that are kinematically and functionally equivalent to their corresponding PGTs but are mathematically non-isomorphic. These PGTs are regarded as isomorphic from a functional point of view [6]. The detection of isomorphism will be greatly complicated or prone to errors if there are pseudo-isomorphic graphs present. As a result, whenever possible, such graphs should be avoided.

Yang et al. [2] introduced a new graph model for representing the structure of PGTs, which had both solid and hollow vertices. Revolute pairs with the same level are equivalent to multiple-joint and are represented by a hollow vertex. Therefore, PGTs with revolute pairs of different levels only have solid vertices. For this graph representation, there exist two different types of graphs; graphs with one or more hollow vertices and graphs without hollow vertex. Since the vertices represent the links, the number of vertices should not exceed the number of links. In a hollow vertex graph, the number of vertices exceeds the number of links, hence, there is no one-to-one correspondence with the PGT.

Links 1, 2, 3, and 4 in the graph depicted in Figure 1 (d) share a common joint axis, "a". Vertex selection is the act of changing a revolute edge with one that is of the same level [7]. Using the vertex selection approach, the graph may be reconfigured to include an articulation point. Hence, the graph represents a fractionated, two-DOF PGM.

The formula for the degree of freedom (F) of a v-vertex graph is

$$F = 3 \times (v - 1) - 2 \times e_r - 1 \times e_g \quad (1)$$

where the number of revolute edges is denoted by $e_r = v - 1$ and the number of geared edges is denoted by $e_g = v - 1 - F$.

For the Simpson PGM shown in Figure 1(d), we have $v = 7$ and $e_r = 6$, $e_g = 4$. Equation (1) gives $F = 3(7 - 1) - 2 \times 6 - 4 = 2$. For the PGT shown in Figure 1(c), we have $v = 6$ and $e_r = 5$, $e_g = 4$. Equation (1) gives $F = 3(6 - 1) - 2 \times 5 - 4 = 1$. Therefore, it is a fractionated 2-DOF PGM. It consists of a 1-DOF PGT that is held up by the frame on a central axis. However, this graph model has trouble accurately modeling a PGM containing multiple joints. This necessitated the use of a graph that is consistent with its mechanism called the rooted graph. The vertex which represents the frame of a mechanism is referred to as the root in a rooted graph representation. Because at least one link in the PGT has its geometric axis rotated around the fixed axis of the mechanism, all graphs must have a root. Figure 1 (f) shows the rooted graph for the Simson PGM. Vertex 1 is the root.

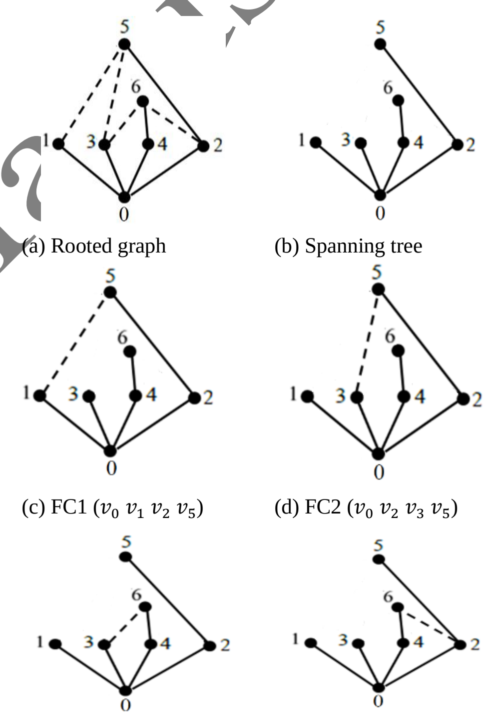
1.3 Literature Review

Methods to identify and remove isomorphic graphs were used during the synthesis of PGTs [1, 2, 8-17]. Two graphs are said to be isomorphic if their edges and vertices maintain adjacency characteristics. Ravisankar and Mruthyunjaya [12] proposed an approach for detecting isomorphism in unlabeled graphs by employing adjacency matrix characteristic coefficients. Rao and Rao [13] identified isomorphic graphs using the Hamming matrix approach and the moment technique. Based on their prior perimeter-loop-based isomorphism identification approach, Yang and Ding [14-16] introduced a fully automated methodology for detecting isomorphic PGTs. Rai and Punjabi [17] described a simple link labeling approach that was utilized to identify a binary sequence that yields the largest binary code. To compare the isomorphism of PGTs, maxi codes are constructed, involving binary code and binary sequence. There are several empirical isomorphism testing methods that rely on a number of distinctive characteristics that, when combined, are sufficient to detect isomorphism [18-23]. However, there is a chance that structural isomorphism will go undetected. Counterexamples have been reported.

2. Fundamental principles

PGMs are distinguished by their unusual kinematic structure, which is comprised solely of revolute and geared joints. A spanning tree is formed by the revolute edges of a rooted graph. The spanning tree is an acyclic subgraph of the original graph, consisting of all of its vertices but no circuits [18]. As shown in Figure 2 (b), a spanning tree can be generated from a rooted graph by simply deleting the geared edges from the graph shown in Figure 2 (a). When a geared edge is added to a spanning tree, a uniquely defined fundamental circuit (FC) is formed. The set of circuits formed by all the geared edges serves as the foundation for the circuit space. The graph of the Simpson gear mechanism includes a total of four FCs. The fundamental circuits all share one geared edge and many revolute edges. They are illustrated in Figure 2, subgraphs (c), (d), (e), and (f).

The degree of a vertex is the number of edges that are incident with it. It is possible to categorize the vertices of a graph according to their vertex degrees.



(a) Rooted graph (b) Spanning tree

(c) FC1 ($v_0\ v_1\ v_2\ v_5$) (d) FC2 ($v_0\ v_2\ v_3\ v_5$)

(e) FC3 ($v_0\ v_3\ v_4\ v_6$) (f) FC4 ($v_0\ v_2\ v_4\ v_6$)

Figure 2. The spanning tree and the FCs of the Simpson gear mechanism.

If d_i is the degree of vertex i, the vertex degree array can be described as a set of numbers that collectively reflect the degrees of the vertices.$[d_0, d_1, d_2, \ldots, d_{v-1}]$.

$$D = [d_0, d_1, d_2, \ldots, d_{v-1}] \quad (2)$$

For example, the vertex degree array for the rooted graph shown in Figure 2 (a) is [4, 2, 3, 3, 2, 3, 3], indicating that there is one vertex of degree four, four vertices of degree three, and two vertices of degree two. The VDA for the spanning tree shown in Figure 2 (b) is [4, 1, 2, 1, 2, 1, 1], where $d_0 = 4$, $d_2 = d_4 = 2$, and $d_1 = d_3 = d_5 = d_6 = 1$.

By assigning a weight of two to geared edges and one to revolute edges, the weighted vertex degree array is produced.

$$D_w = [d_{0w}, d_{1w}, d_{2w}, \dots, d_{(v-1)w}] \quad (3)$$

In Fig. 2 (a), for instance, edges e_{03}, e_{35}, and e_{36} are incident with vertex 3. The three edges have respective weights of 1, 2, and 2. Therefore, the weighted vertex degree of vertex 3 is 2 + 1 + 2 = 5 and the array of weighted vertex degrees for geared graph is [4, 3, 4, 5, 2, 5, 5].

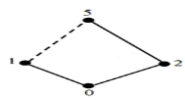


Figure 3. FC1 after being separated from the geared graph shown in Figure.

The weighted vertex degree array for the separated FC1 is [3, 3, 2, 2], indicating two two-degree vertices and two three-degree vertices.

3. Spanning Tree Classification

Using the rooted graph, it is possible to differentiate explicitly the similarities and differences between numerous PGMs. Specifically, the vertices may be subdivided into many levels. The root is located at the ground level. First-level vertex refers to a vertex that has a direct connection to the root by a single revolute edge. If a vertex has two revolute edges connecting it to its root, it is considered to be of the second level. This may be repeated on subsequent levels, if applicable.

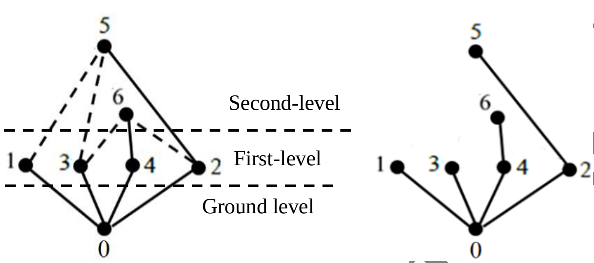


Figure 4. vertex levels.

Graphs may be grouped into families. Graphs may be classified into families based on their spanning trees. Graphs from distinct families cannot beisomorphic.Two graphswith different spanning trees are not isomorphic .The vertex degree string is used to classify spanning tree. It is defined as an ascending sequence of numbers denoting the degree of vertices beginning from the ground level. In particular, the first number in the vertex degree string denotes the degree of the ground vertex, the second denotes the degree of the vertex with the highest vertex degree in the first level, and so on. For example, the spanning tree shown in Fig. 4 (b) has a vertex degree string of 4221111.

4. Matrix Representation

In order to facilitate computer programming, the graph of a PGM is expressed as a matrix.

The vertex-circuit matrix, C_v, is defined as :

$$C_v = \begin{bmatrix} & & circuit\ l & & \\ vertex\ v & c_{1,1} & c_{1,2} & \cdots & c_{1,l} \\ & c_{2,1} & c_{2,2} & \cdots & c_{2,l} \\ & \vdots & \vdots & \vdots & \vdots \\ & c_{v,1} & c_{v,2} & \cdots & c_{v,l} \end{bmatrix} \quad (4)$$

Where:

$$C_v(v,l) = \begin{cases} 1 & if\ vertex\ v\ is\ a\ vertex\ of\ circuit\ l, \\ 0 & otherwise. \end{cases}$$

If e is the number of edges in a rooted graph, v the number of vertices, then C_v is a $v \times e_g$ matrix, because the number of fundamental circuits is e_g, each fundamental circuit is produced by one geared edge.

As illustrated in Figure 2 (a), the fundamental circuits are : FC1 ($v_0\ v_1\ v_2\ v_5$), FC2 ($v_0\ v_2\ v_3\ v_5$), FC3 ($v_0\ v_3\ v_4\ v_6$), and FC4 ($v_0\ v_2\ v_4\ v_6$). Therefore the C_v matrix is :

$$C_v = \begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \quad (5)$$

The weighted vertex-circuit matrix, $[C_v]_w$, can be obtained from the C_v matrix by giving each vertex its fundamental circuit weight. We obtain

$$[C_v]_w = \begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 2 & 2 & 2 & 2 \\ 3 & 0 & 0 & 0 \\ 2 & 2 & 0 & 3 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 2 & 2 \\ 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix} \end{array} \quad (6)$$

5. Detection of Isomorphism

The majority of earlier approaches for establishing structural isomorphism rely on incidence matrices. Nonetheless, the labeling of links affects the form of the incidence matrix. Relabeling the vertices of Figure 2 (a), for instance, yields the graph illustrated in Figure 5.

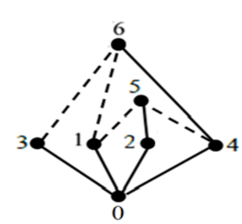
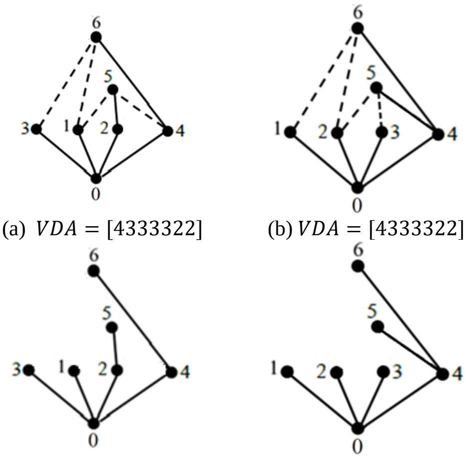


Figure 5. The Simpson gear train graph with relabeled vertices.

The graphs in Figs. 2(a) and 5 show identical gear trains, but their incidence matrices are different owing to vertex labeling. However, the weighted vertex degree string of the graph is not affected by labeling of the vertices. Fig. 6 (a) and (b) show different gear trains because of their spanning trees.



(a) $VDA = [4333322]$ (b) $VDA = [4333322]$

(c) $VDA = [4221111]$ (d) $VDA = [4311111]$

Figure 6. Isomorphic identification by the vertex degree strings of spanning trees.

The vertex degree string (VDS) for the spanning tree in Fig. 6 (c) is 4221111, whereas it is 4311111 for the tree in Fig. 6 (d). Since $VDS_c \neq VDS_d$, the two graphs are **not** isomorphic. Isomorphism cannot exist between rooted graphs with distinct spanning trees (vertex degree strings). The first step in identifying isomorphism is to classify graphs depending on their VDSs. Isomorphism is not possible in graphs with different VDSs. The required condition for testing graph isomorphism is provided by the VDSs of spanning trees. When there is no one-to-one correspondence between the vertices of two graphs, we say that they are not isomorphic, and we test this by comparing their correspondence. Therefore, the adjacency features of graphs allow for the identification of structural isomorphism both graphically and numerically. The characteristics of a graph's adjacencies are established by

1.the spanning tree embedded in the graph
2.the relative position of the fundamental circuits in the graph.

This can be done by multiplying the weighted vertex degree array D_w by the weighted vertex-circuit matrix, $[C_v]_w$. This produces one solution called the fundamental circuit assortment array (FCAA).

$$FCAA = D_w\,[C_v]_w \tag{7}$$

Regardless of vertex labeling of the geared graph shown in Fig. 2 (a), similar FCAAs are generated. This reduces the structural isomorphism test to a comparison of the FCAAs of the two graphs under consideration. From Eq. (7), the FCAA for the graph shown in Figure 2 (a) is :

$$FCAA = [4, 3, 4, 5, 2, 5, 5]\begin{bmatrix} 2 & 2 & 2 & 2 \\ 3 & 0 & 0 & 0 \\ 2 & 2 & 0 & 3 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 2 & 2 \\ 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 40 \\ 46 \\ 42 \\ 39 \end{bmatrix} \tag{8}$$

Arrange the FCAA in a descending order of circuit degrees to obtain the circuit degree string. There are 4 circuit degrees in the FCAA, namely 40, 46, 42, and 39. By putting these degrees in order, we get a circuit degree string of 46424039. It can be written in sequence with the vertex degree string of the spanning tree to obtain the isomorphic identification number (IIN)

$$\text{IIN} = [422111146424039] \tag{9}$$

From Eq. (7), the FCAA for the graph shown in Figure 5 is :

$$FCAA^{*} = [4, 5, 2, 3, 4, 5, 5]\begin{bmatrix} 2 & 2 & 2 & 2 \\ 3 & 0 & 3 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 2 & 3 & 0 & 2 \\ 0 & 3 & 3 & 0 \\ 3 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 46 \\ 39 \\ 42 \\ 40 \end{bmatrix} \tag{10}$$

The isomorphic identification number is

$$IIN^{*} = [422111146424039] \tag{11}$$

Since $IIN = IIN^{*}$, the two graphs are isomorphic.

6. Algorithm

Using spanning trees and the fundamental circuits of kinematic chains, this work presents a method for isomorphic identification, which is described in detail below.

1.Determine the degree of each vertex in the spanning tree, then sort the vertices in decreasing order of vertex degree, starting at the first level.

2.Determine whether the two graphs are isomorphic by comparing the VDSs of their spanning trees. If they are identical, continue to step 2.

3.Renumber the vertices according to the vertex degrees of the spanning tree.

4.Assign weights to the edges of the graph, and then locate the weighted vertex degree array (WVDA).

5.Calculate the fundamental circuit assortment array (FCAA).

6.Arrange the FCAA in decreasing order of circuit degrees.

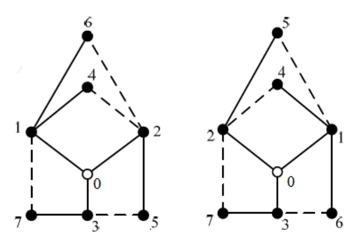
7.Write the circuit degree string in sequence with the vertex degree string of the spanning tree to obtain the isomorphic identification number.

8.For two graphs to be isomorphic, their isomorphic identification numbers must be the same.

7. Validation and Discussion

7.1 Case study 1

Figure 7 depicts the rooted graphs of two 3-DOF, 8-link PGTs. Both are discussed in references [14 and 24].



(a) (b)

Figure 7. Two 8-link 3-DOF PGTs

The vertex degree string for both of the two graphs in Fig. 7 is [3322111. Therefore, it appears likely that they are isomorphic. It should be emphasized that the vertex degree string is a required but not sufficient requirement for isomorphism. The weighted vertex degree arrays of the two graphs shown in Fig. 7 are $(D_w)_a = [3\;5\;6\;4\;3\;3\;3\;3]$ and $(D_w)_b = [3\;5\;6\;4\;3\;3\;3\;3]$.

From Eq. (7), the fundamental circuit assortment array (FCAA) for Fig. 7 (a) is

$$(FCAA)_a = [3\;5\;6\;4\;3\;3\;3\;3]\begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 3 & 0 \\ 3 & 3 & 0 & 2 \\ 0 & 0 & 2 & 3 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 43 \\ 43 \\ 38 \\ 39 \end{bmatrix} \tag{12}$$

Therefore, the isomorphic identification number is

$$(IIN)_a = [3322111143433938] \tag{13}$$

For Fig. 7 (b), the FCAA is

$$(FCAA)_a = [3\;5\;6\;4\;3\;3\;3\;3]\begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 0 & 2 \\ 3 & 3 & 3 & 0 \\ 0 & 0 & 2 & 3 \\ 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 43 \\ 43 \\ 41 \\ 37 \end{bmatrix} \tag{14}$$

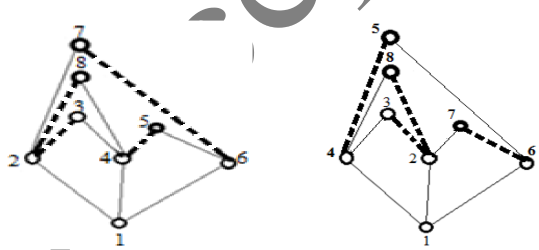
Therefore, the isomorphic identification number is

$$(IIN)_b = [3322111143434137] \tag{15}$$

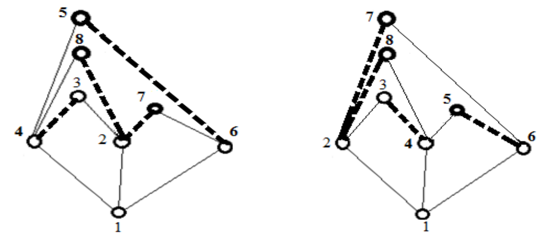
The PGTs depicted in Fig. 7 are non-isomorphic because $(IIN)_a \neq (IIN)_b$. References [14 and 24] confirms this finding.

7.2 Case study 2

Figure 8 depicts four different graphs, each of which have the same spanning tree VDA [3 3 2 2 1 1 1 1]. However, isomorphic possibilities exist, as will be shown in the following explanation.



(a) $(IIN)_a =$ 3322111143433938 (b) $(IIN)_b =$ 3322111143433938



(c) $(IIN)_c =$ 3322111143424137 (d) $(IIN)_d =$ 3322111143424137

Figure 8. Graphs sharing the same spanning tree VDA.

The isomorphic identification numbers for the four graphs shown in Fig. 8 are $(IIN)_a = 3322111143433938$, $(IIN)_b = 3322111143433938$, $(IIN)_c = 3322111143424137$, and $(IIN)_d = 3322111143424137$, respectively. $(IIN)_a = (IIN)_b$, and $(IIN)_c = (IIN)_d$. Consequently, the graphs in Figures 8 (a) and (b) are isomorphic. Also, those shown in (c) and (d) are isomorphic. These findings are supported by references [14, 24].

The new isomorphic detection approach is used to a 2-DOF PGM atlas with six links from the literature. Appendix A shows the geared graphs and the isomorphic identification numbers of the 6-link 2-DOF PGMs. The detection results match the those reported by Refs. [25-27] and each graph has a unique isomorphic identification number.

7.3 Case study 3

Figure 9 depicts the graphs of three 2-DOF planetary geared cam mechanisms with 5 links. The vertex degree string for each of the three graphs is 32111. By assigning a weight of one to revolute edges (thin lines), two to geared edges (dotted lines) and three to cam edges (bold lines), the weighted vertex degree arrays of the three graphs are $(D_w)_a = [3\;5\;4\;3\;3]$, $(D_w)_b = [3\;2\;4\;6\;3]$, and $(D_w)_c = [3\;2\;6\;4\;3]$.

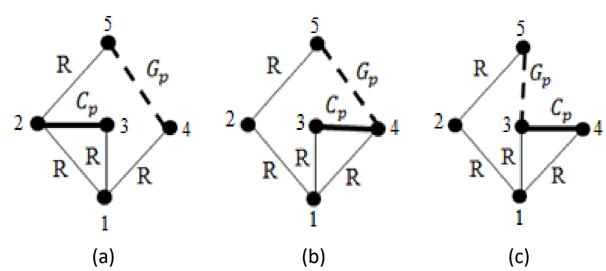


Figure 9. Graphs of three 2-DOF PGCMs with 5 links.

The vertex-circuit matrix for Fig. 9 (a) is,

$$[C_v]_{wa} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{cc} c_1 & c_2 \\ 2 & 2 \\ 2 & 4 \\ 0 & 4 \\ 3 & 0 \\ 3 & 0 \end{array} \tag{16}$$

Therefore, From Eq. (7), the fundamental circuit assortment array (FCAA) is

$$(FCAA)_a = [3\ 5\ 4\ 3\ 3]\begin{bmatrix} 2 & 2 \\ 2 & 4 \\ 0 & 4 \\ 3 & 0 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 34 \\ 42 \end{bmatrix} \tag{17}$$

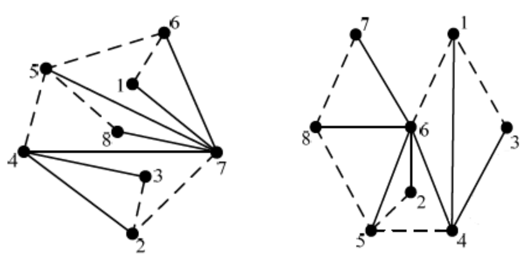
The isomorphic identification number is

$$(IIN)_a = 321114234 \tag{18}$$

Similarly, $(IIN)_b = (IIN)_c = 321114637$. Consequently, the graphs shown in Figures 9 (b) and (c) are isomorphic. Appendix B shows the graphs of 5-link 2-DOF planetary geared cam mechanisms and their corresponding isomorphic identification numbers. Each graph has a unique isomorphic identification number. Isomorphic graphs (not shown) have the same value of isomorphic identification number.

8. Application of the new method to other graph representations

Figure 10 depicts the graphs of two 1-DOF PGTs with 8 links as represented by reference [14]. There are no hollow vertices (roots) in these graphs



(a) (b)

Figure 10. Graphs of two 1-DOF PGTs with 8 links from reference [14].

The vertex degree string for both of the two graphs is 52111111. The weighted vertex degree arrays of the two graphs are $(D_w)_a = [3\ 5\ 3\ 5\ 7\ 5\ 7\ 3]$ and $(D_w)_b = [5\ 3\ 3\ 5\ 7\ 7\ 3\ 5]$. The vertex-circuit matrices are,

$$[C_v]_{wa} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array} \\ \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 2 & 2 \\ 0 & 3 & 3 & 3 & 0 & 0 \\ 3 & 3 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 0 & 3 \\ 0 & 0 & 3 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{19}$$

and

$$[C_v]_{wb} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array} \\ \begin{bmatrix} 3 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 3 & 0 \\ 0 & 3 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 3 \end{bmatrix} \end{array} \tag{20}$$

Therefore, the fundamental circuit assortment array (FCAA) for Fig. 10 (a) is

$$(FCAA)_a = [3\ 5\ 3\ 5\ 7\ 5\ 7\ 3]\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 2 & 2 \\ 0 & 3 & 3 & 3 & 0 & 0 \\ 3 & 3 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 0 & 3 \\ 0 & 0 & 3 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 38 \\ 50 \\ 44 \\ 50 \\ 34 \\ 46 \end{bmatrix} \tag{21}$$

The isomorphic identification number is

$$(IIN)_a = 52111111505046443834 \tag{22}$$

The fundamental circuit assortment array (FCAA) for Fig. 10 (b) is

$$(FCAA)_b = [5\ 3\ 3\ 5\ 7\ 7\ 3\ 5]\begin{bmatrix} 3 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 3 & 0 \\ 0 & 3 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 34 \\ 46 \\ 44 \\ 50 \\ 50 \\ 38 \end{bmatrix} \tag{23}$$

The isomorphic identification number is

$$(IIN)_b = 52111111505046443834 \tag{24}$$

The graphs in Figure 9 are isomorphic because $(IIN)_a = (IIN)_b$. Yang et al. came at the same result. [14] .

9. Conclusions

In this paper, an algebraic method for identifying isomorphism in PGMs and PGCMs is presented, using spanning trees and the fundamental circuits of kinematic chains. The first step in identifying isomorphism is to classify graphs depending on their vertex degree strings. The vertex degree strings of spanning trees give the required condition for checking graph isomorphism. The vertices of two graphs are not isomorphic if their vertex degree strings are not identical. The main advantage of classifying kinematic chains according to their vertex degree strings is that it automatically removes the large majority of isomorphic topologies. This saves time and effort compared to other approaches. In the second step, a unique identifier capable of detecting isomorphism is developed and the task of evaluating structural isomorphism changes into a problem involving the examination of the isomorphic numbers of the two PGTs in question. The isomorphic identifier is calculated from the vertex degree array and the circuit matrix through matrix operations. Each graph has a unique isomorphic identification number. There is agreement between the detection results and the findings given in Refs. [25-27]. Compact notations, outstanding accuracy, and simplicity of usage are the key benefits of this system. Additionally, it is mathematically simple and can be applied to every type of graph representation.

Authors' contribution

All authors contributed equally to the preparation of this article.

Declaration of competing interest

The authors declare no conflicts of interest.

Funding source

This study didn't receive any specific funds.

REFERENCES

[1] Buchsbaum, F., and Freudenstein, F., 1970, "Synthesis of Kinematic Structure of Geared Kinematic Chains and other Mechanisms," J. Mech., 5(3), pp. 357-392.doi.org/10.1016/0022-2569(70)90068-6

[2] Yang, W. J., Ding, H. F., Zi, B., and Zhang, D., 2018, "New Graph Representation for Planetary Gear Trains," ASME J. Mech. Des., 140 (1), p. 012303.doi.org/10.1016/j.aej.2021.09.036

[3] Kamesh,V. V., Rao, K. M., and Rao, A. B. S., 2017, "An Innovative Approach to Detect Isomorphism in Planar and Geared Kinematic Chains Using Graph Theory," ASME J. Mech. Des., 139(12), p.122301.doi.org/10.1177/0954406221998399

[4] M. Du , L. Yang , A basis for the computer-aided design of the topological structure of planetary gear trains, Mech. Mach. Theory 145 (2020) 103690 .DOI:10.1016/j.mechmachtheory.2019.103690

[5] V.R. Shanmukhasundaram, Y.V.D. Rao, S.P. Regalla, Review of structural synthesis algorithms for epicyclic gear trains. In: D. Sen, S. Mohan, G. Ananthasuresh (eds), Mechanism and Machine Science. Lecture Notes in Mechanical Engineering. Springer, Singapore, 2021, https://doi.org/10.1007/ 978- 981- 15- 4477- 4 _ 25 .

[6] Tsai LW(2000) Mechanism design: enumeration of kinematic structures according to function. CRC Press, Florida, USA.

[7] Kamesh, Vinjamuri Venkata; Mallikarjuna Rao, Kuchibhotla; Srinivasa Rao, Annambhotla Balaji (2017). Topological Synthesis of Epicyclic Gear Trains Using Vertex Incidence Polynomial. Journal of Mechanical Design, 139(6), 062304–. doi:10.1115/1.4036306. doi.org/10.1115/1.4036306

[8] Freudenstein, F., An application of Boolean algebra to the motion of epicyclic drives, ASME J. Eng. Ind. 93 (1), 1971, 176–182. doi.org/10.1115/1.3427871

[9] Yang, W.J. , Ding, H.F., Kecskeméthy, A., Automatic Structural Synthesis of Non-Fractionated 2-DOF Planetary Gear Trains, Mech. Mach.Theory155,2021,104125-1-27. DOI:10.1016/j.mechmachtheory.2020.104125

[10] Shanmukhasundaram, V.R., Rao, Y.V.D. , Regalla, S.P., Enumeration of displacement graphs of epicyclic gear train from a given rotation graph using concept of building of kinematic units, Mech. Mach. Theory 134, 2019, 393–424. doi.org/10.1016/j.mechmachtheory.2019.01.005

[11] Hsu, C. H., Lam, K. T., and Yin, Y. L., Automatic Synthesis of Displacement Graphs for Planetary Gear Trains," Mathl. Comput. Modelling, 19(11), 1994, 67-81. doi /10.22055/JACM.2022.41255.3721

[12] Ravisankar. R., Mruthyunjaya. T. S., 1985, "Computerized synthesis of the structure of geared kinematic chains", Mechanism and Machine Theory, 20, pp. 367-387. DOI: 10.1016/0094-114X (85)90042-4

[13] Rao, Y. V. D., Rao, A. C., Generation of Epicyclic Gear Trains of One Degree of Freedom," ASME J. Mech. Des., 130(5), (2008) pp. 052604. doi.org/10.1115/1.2890107

[14] Yang, W. and Ding, H.: The Perimeter Loop-Based Method for the Automatic Isomorphism Detection in Planetary Gear Trains, J. Mech. Design, 140, 1–10, 2018. doi.org/10.1115/1.4041572

[15] Yang, W. and Ding, H.: Automatic detection of degenerate planetary gear trains with different degree of freedoms, Appl. Math. Model., 64, 320–332, 2018. DOI:10.1016/j.apm.2018.07.038

[16] Yang, W. and Ding, H.: The Complete Set of One- Degree-of- Freedom Planetary Gear Trains With Up to Nine Links, J. Mech. Design, 141, 1–22, 2019. doi.org/10.1115/1.4041482

[17] Rai, R. K. and Punjabi, S.: A new algorithm of links labelling for the isomorphism detection of various kinematic chains using binary code, Mech. Mach. Theory, 131, 1–32, 2019. DOI:10.1016/j.mechmachtheory.2018.09.010

[18] Yi H J, Wang J P, Hu Y L and Yang P 2021 Mechanism isomorphism identification based on artificial fish swarm algorithm. P. I. Mech. Eng. C-J. Mech. 235(21): 5421–5433. DOI:10.1177/0954406220977560

[19] Zou Y H and He P 2016 An algorithm for identifying the isomorphism of planar multiple joint and gear train kinematic chains. Math. Probl. Eng. 2016: 5310582. doi.org/10.1155/2016/5310582

[20] Huang P, Liu T T, Ding H F and Zhao Y Q 2021 Isomorphism identification algorithm and database generation for planar 2–6 DOFs fractionated kinematic chains combined by two or three non-fractionated kinematic chains. Mech. Mach. Theory 166: 104520. doi.org/10.1007/s12046-022-01918-y

[21] He L Y, Liu F X, Sun L and Wu C Y 2019 Isomorphic identification for kinematic chains using variable high-order adjacency link values. J. Mech. Sci. Technol. 33(10): 4899–4907. https://doi.org/10.1007/s12206-019-0930-9

[22] Rai R K and Punjabi S 2018 Kinematic chains isomorphism identification using link connectivity number and entropy neglecting tolerance and clearance. Mech. Mach. Theory 123: 40–65. doi.org/10.1177/16878132221131193

[23] Kamesh V V, Rao K M and Rao A B S 2017 An innovative approach to detect isomorphism in planar and geared kinematic chains using graph theory. J. Mech. Des. 139(12): 122301. DOI:10.1115/1.4037628

[24] Hsu, C. H., 1994, "Displacement Isomorphism of Planetary Gear Trains," Mech. Mach. Theory, 29(4), pp. 513-523. doi.org/10.1016/0094-114X(94)90091-4

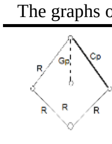
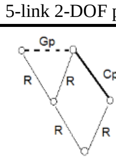
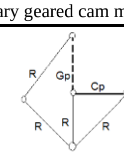
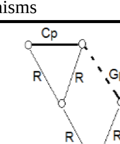
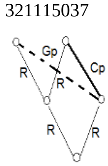
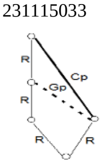
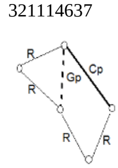
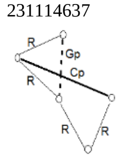
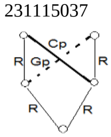
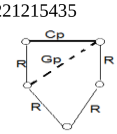
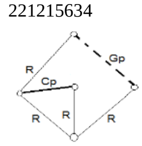
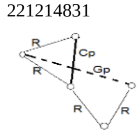
[25] Hsu, C. H., and Hsu, J. J., 1997, "An Efficient Methodology for the Structural Synthesis of Geared Kinematic Chains," Mech. Mach. Theory, 32(8), pp. 957-973. doi.org/10.1177/0954406215583321

[26] Prasad Raju Pathapati, V. V. N. R., and Rao, A. C., "A New Technique Based on Loops to Investigate Displacement Isomorphism in Planetary Gear Trains," ASME J. Mech. Des., 2002, 124(4), pp. 662-675. doi: https://doi.org/10.1115/1.1519276

[27] Hind A. Nafeh, Essam L. Esmail, Sajad H. Abdali, Automatic Structural Synthesis of Planetary Geared Mechanisms using Graph Theory, J. Appl. Comput. Mech., Volume 9, Issue 2, April 2023, Pages 384-403. 10.22055/JACM.2022.41255.3721

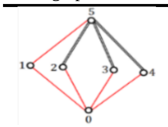
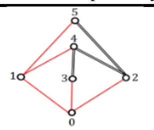
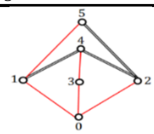
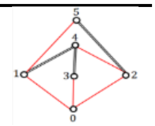
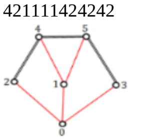
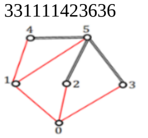
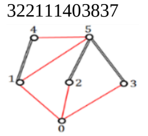
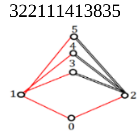
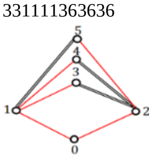
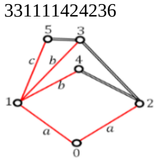
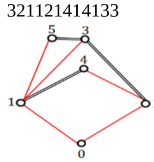
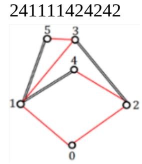
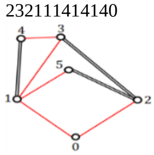
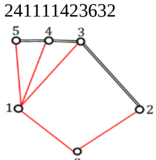
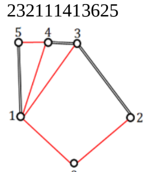
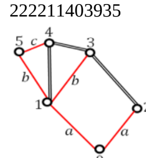
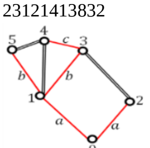
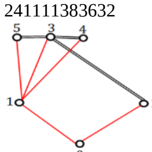
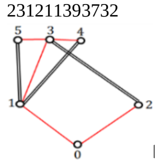
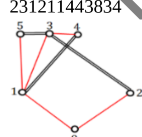
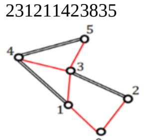
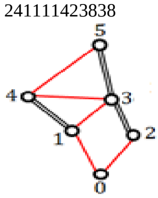
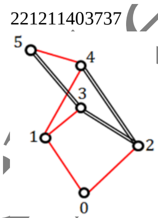
Appendix A

Table A The graphs of the 6-link 2-DOF PGMs and their correspondingisomorphic identification numbers. The IDs are produced by giving revolute edges (shown in red) a weight of one and geared edges (shown in black) a weight of two.

The graphs of the 5-link 2-DOF planetary geared cam mechanisms			
			
321115037	231115033	321114637	231114637
			
231115037	221215435	221215634	221214831
			
222114835	222115638	321114234	221214435

Appendix B

Table B The graphs of the 5-link 2-DOF planetary geared cam mechanisms and their corresponding isomorphic identification numbers. In order to generate the IDs, revolute edges (thin lines) are given a value of one, geared edges (dotted lines) are given a value of two, and cam edges (bold lines) are given a value of three.

The graphs of the 6-link 2-DOF planetary gear mechanisms			
			
421111424242	331111423636	322111403837	322111413835
			
331111363636	331111424236	321121414133	241111424242
			
232111414140	241111423632	232111413625	222211403935
			
23121413832	241111383632	231211393732	231211443834
			
231211423835	241111423838	221211403737	231211413735
			
221311373634	221221393635	231211403837	